COSMOS/M

A COMPLETE FINITE ELEMENT ANALYSIS SYSTEM

2.7

Basic System Finite Element Analysis
Part 2

STRUCTURAL RESEARCH & ANALYSIS CORP.

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1 About the Verification Problems...

Introduction

COSMOS/M software modules are continually in the process of extensive development, testing, and quality assurance checks. New features and capabilities incorporated into the system are rigorously tested using verification examples and in-house quality assurance problems. All verification problems are provided to the user along with the software, and they are made available in the COSMMOS/M directory. There are more than 150 verification problems for analysis modules in the Basic System.

The purpose of this section is dual fold: to present many example problems that test a combination of capabilities offered in the COSMOS/M Basic System, and to provide a large number of verification problems that validate the basic modeling and analysis features. The first part of this manual presented several fully described and illustrated examples which cover few aspects of modeling and analysis limitations. This part provides examples on many other analysis features of the Basic System.

The input files for all verification problems are provided in separate folders (depending on the analysis type) in the "...\Vprobs" directory where "..." denotes the COSMOS/M directory.

Folder	Analysis Type
Geostar	GEOSTAR modeling examples
Buckling	***Linearized buckling analysis***
AdvDynamics	Linear dynamic response analysis
Emagnetic	Electromagnetic analysis
Frequency	***Frequency (modal) analysis***
Fatigue	Fatigue analysis
Nonlinear	Nonlinear dynamic analysis
Static	***Linear static stress analysis***
Thermal	Thermal (heat transfer) analysis (linear)
FFE	FFE modules
HFS	High frequency electromagnetic simulation

To use the verification problems, enter GEOSTAR and at the GEO> prompt, execute the command **Load... (FILE)** from the File menu. The following pages show a listing of the verification problems based on analysis and element type's.

Classification by Analysis Type

Analysis Type	Folder	Problem Title
Linear Static Analysis	...\Vprobs\Static	S1, S2, S3A, S3B, S4, S5, S6, S7, S8, S9A, S9B,S10A, S10B, S11, S12, S13, S14A,S14B, S15, S16A, S16B, S17, S18, S19, S20, S21A, S21B, S22, S23, S24, S25, S26, S27, S28, S29, S29A, S30, S31, S31A, S32A, S32B, S32C, S32D, S32M, S33A, S33M, S34, S35A, S35B, S36A, S36M, S37, S38, S39, S40, S41, S42, S43, S44A, S44B, S45, S46, S46A, S46B, S47, S47A, S47B, S48, S49A, S49B, S50A, S50B, S50C, S50D, S50F, S50G, S50H, S51, S52, S53, S54, S55, S56, S57, S58, S58B, S59A, S59B, S59C, S60, S61, S62, S63, S64A, S64B, S65, S66, S67, S68, S69, S70, S71, S74, S75, S76, S77, S78, S79, S80, S81, S82, S83, S84, S85, S86, S87,
Buckling Analysis	...\Vprobs\Buckling	B1, B2, B3, B4, B5A, B5B, B6, B7A, B7B, B8, B9, B10, B11, B12, B13, B14, B15A, B15B
Modal Analysis	...\Vprobs\Frequency	F1, F2, F3, F4, F5, F6, F7, F8, F9, F10, F11A, F11B, F12, F13, F14, F16A, F16B, F17, F18, F19, F20A, F20B, F20C, F20D, F20, F20E, F20G, F20F, F21, F22, F23, F24, F25, F26, F27, F28

Classification by Element Type

Element Name	Analysis Type	Problem Title
BEAM2D	Buckling Linear Static	B10, B11, B13 S9A, S9B, S24, S41, S46, S47, S51, S52, S53, S54, S75, S76
BEAM3D	Buckling Modal Analysis Linear Static	B1, B2, B3, B6, B9 F3, F4, F5, F12, F17 S7, S22, S23, S26, S27, S28, S33A, S33M, S34, S39, S43, S45, S55
BOUND	Linear Static	NONE
ELBOW	Linear Static	S15, S16A, S16B
GAP	Linear Static	S75, S76
GENSTIF	All	NONE
MASS	Modal Analysis Linear Static	F1, F5, F6 S39
PIPE	Modal Analysis Linear Static	F6 S16A, S16B
PLANE2D	Modal Analysis Linear Static	F2, F20A, F20B, F21, F23 S2, S5, S6, S17, S19, S38, S46, S46A, S48, S49A, S50A, S50B, S50C, S61, S62, S65, S66, S67, S68, S70, S76, S82, S83, S86
RBAR	Linear Static	F28
SHELL3	Buckling Modal Analysis Linear Static	B5 F11 S3A, S3B, S8, S30, S33A
SHELL3L	Linear Static	NONE
SHELL3T	Modal Analysis	F8, F16A
SHELL4	Buckling Modal Analysis Linear Static	B4, B7, B9 F7, F9, F10, F18, F27A, F27B S20, S25, S33A, S33M, S36A, S36M, S42, S44A, S44B, S85
SHELL4L	Linear Static	S21A, S31, S43, S59B, S71

Classification by Element Type (Concluded))

Element Name	Analysis Type	Problem Title
SHELL4T	Modal Analysis Linear Static	F16B S50C
SHELL9	Linear Static	S46B, S56, S57, S58, S59A, S60
SHELL9L	Linear Static	S21B, S29A, S31A, S59A
SHELLAX	Buckling Modal Analysis Linear Static	B8, B14, B15 F19, F25, F26 S18, S37, S79, S80, S81
SOLID	Modal Analysis Linear Static	F13, F20E, F20F, F22 S10A, S10B, S11, S35A, S47, S47B, S49B, S50F, S50G, S77
SHELL6	Buckling Modal Analysis Linear Static	B5B, B7B F7B, F11B, F20H S6B, S20B, S42B, S50I
SOLIDL	Linear Static	S29, S35B, S59C
SOLIDPZ	Modal Analysis	F24
SPRING	Modal Analysis	NONE
TETRA10	Modal Analysis Linear Static	F20D S50E
TETRA4	Linear Static	NONE
TETRA4R	Linear Static Modal Analysis	S50H, S58B, S74 F20G
TRIANG	Modal Analysis Linear Static	F20C S50D, S64A, S64B, S68, S69, S78, S84
TRUSS2D	Buckling Modal Analysis Linear Static	B6 F14 S4, S32A, S32B, S32C, S32D, S32M, S40, S63, S76
TRUSS3D	Modal Analysis Linear Static	F1 S1, S12, S13, S14A, S14B, S22, S26, S33A, S33M

2 Linear Static Analysis

Introduction

This chapter contains verification problems to demonstrate the accuracy of the Linear Static Analysis module STAR.

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S1: Pin Jointed Truss

TYPE:

Static analysis, truss element (TRUSS3D).

REFERENCE:

Beer, F. P., and Johnston, E. R., Jr., "Vector Mechanics for Engineers: Statics and Dynamics," McGraw-Hill Book Co., Inc. New York, 1962, p. 47.

PROBLEM:

A 50 lb load is supported by three bars which are attached to a ceiling as shown. Determine the stress in each bar.

GIVEN:

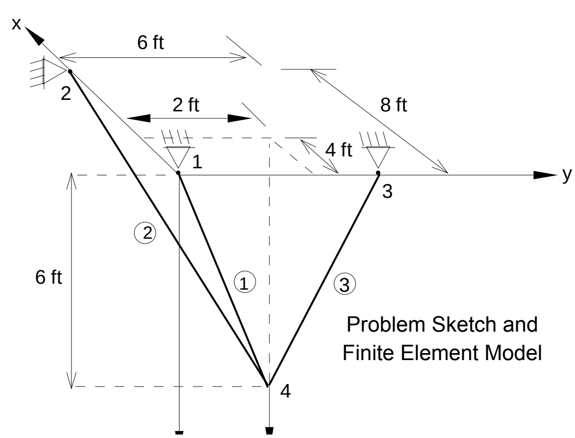
Area of each bar = 1 in^2

E = 30 x 10^6 psi

COMPARISON OF RESULTS

	σ_{1-4}, psi	σ_{2-4}, psi	σ_{3-4}, psi
Theory	10.40	31.20	22.90
COSMOS/M	10.39	31.18	22.91

Figure S1-1



S2: Long Thick-Walled Cylinder

TYPE:

Static analysis, 2D axisymmetric elements (PLANE2D).

REFERENCE:

Timoshenko, S. P. and Goodier, J., “Theory of Elasticity,” McGraw-Hill, New York, 1951, pp. 58-60.

PROBLEM:

Calculate the radial stresses for an infinitely long, thick walled cylinder subjected to an internal pressure p.

GIVEN:

a = 100 in

b = 115 in

p = 1000 psi

E = 30 x 10^6 psi

ν = 0.3

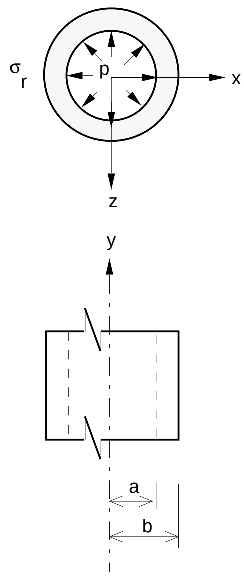
MODELING HINTS:

The model is meshed with three elements through the thickness and three elements along the length.

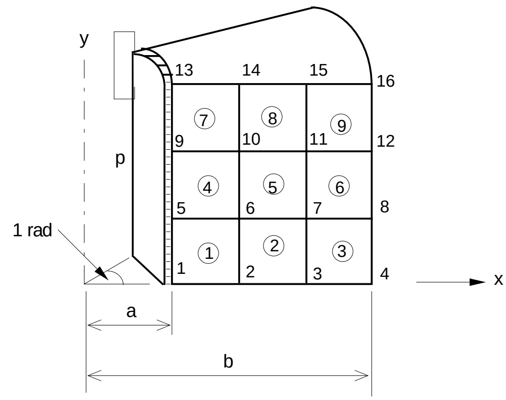
COMPARISON OF RESULTS:

r (Radial Distance) (in)	Radial Stress σ_r (psi)	
	Theory	COSMOS/M
102.5 (Element 1)	-802.40	-802.51
107.5 (Element 2)	-447.75	-447.84
112.5 (Element 3)	-139.34	-139.42

Figure S2-1



Problem Sketch



Finite Element Model

S3A, S3B: Simply Supported Rectangular Plate

TYPE:

Static analysis, 3-node thin plate element (SHELL3).

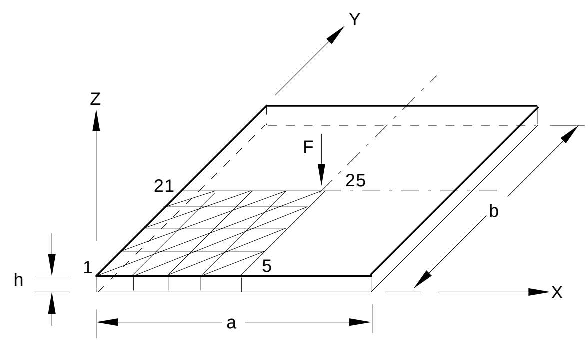
REFERENCE:

Timoshenko, S. P. and Woinowsky-Krieger, “Theory of Plates and Shells,” McGraw-Hill Book Co., 2nd edition. pp. 143-120, 1962.

PROBLEM:

Calculate the deflection at the center of a simply supported isotropic plate subjected to (A) concentrated load F, (B) uniform pressure (P).

Figure S3-1



Problem Sketch and Finite Element Model

GIVEN:

E = 30,000,000 psi
ν = 0.3
h = 1 in
a = b = 40 in
F = 400 lb
p = 1 psi

MODELING HINTS:

Due to double symmetry in geometry and loads, only a quarter of the plate is modeled.

COMPARISON OF RESULTS:

Case	X (in)	Y (in)	Deflection at Node 25 (UZ)	
			Theory	COSMOS/M
A	20	20	-0.0270230 in	-0.027123 in
B	20	20	-0.00378327 in	-0.0037915 in

S4: Thermal Stress Analysis of a Truss Structure

TYPE:

Linear thermal stress analysis, truss elements (TRUSS2D).

REFERENCE:

Hsieh, Y. Y. "Elementary Theory of Structures," Prentice-Hall, Inc., 1970, pp. 200-202.

PROBLEM:

Determine the member forces in truss stucture shown in the figure subject to a 50° F rise in temperature at the top chords (elements 13 and 14).

GIVEN:

$E = 30 \times 10^6$ psi

Coefficient of thermal expansion = $\alpha = 0.65 \times 10^{-5}$/° F

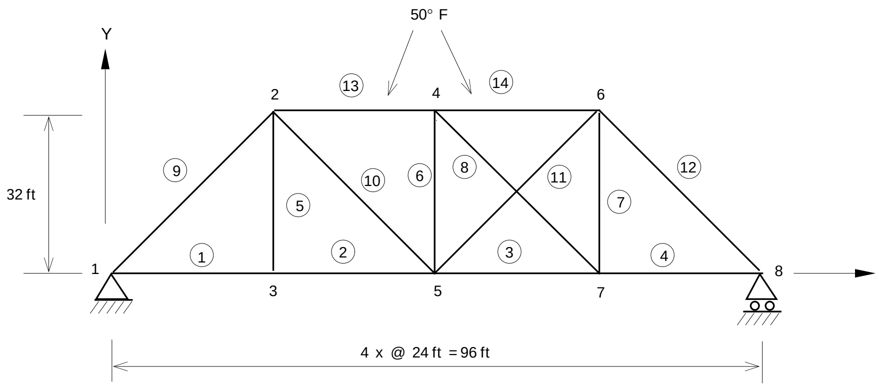
L(ft)/A(in^2) = 1for all members

COMPARISON OF RESULTS:

Member Forces (kips					
Members	**Theory**	**COSMOS/M**	**Members**	**Theory**	**COSMOS/M**
1	0	0	8	35.1	35.1
2	0	0	9	0	0
3	-21.1	-21.1	10	0	0
4	0	0	11	35.1	35.1
5	0	0	12	0	0
6	-28.1	-28.1	13	0	0
7	-28.1	-28.1	14	-21.1	-21.1

COSMOS/M results are calculated by listing element stress results and multiplying by the corresponding area.

Figure S4-1



Problem Sketch and Finite Element Model

S5: Thermal Stress Analysis of a 2D Structure

TYPE:

Linear thermal stress analysis, 2D elements (plane strain, PLANE2D).

PROBLEM:

Determine the displacements and stresses of the plane strain problem shown in figure due to a uniform temperature rise.

GIVEN:

E = 30 x 10^6 psi

α = 0.65 x 10^{-5}/° F

ν = 0.25

T = 100° F

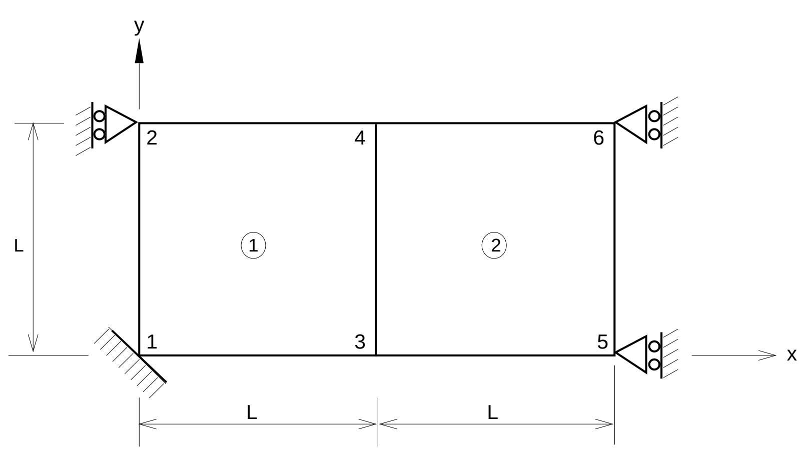
L = 1 in

COMPARISON OF RESULTS:

Displacements at Nodes (2, 4, and 6)

	Y-Displacement (in)	SX-Stress (psi)
Theory	0.001083	-26000.0
COSMOS/M	0.001083	-26000.1

Figure S5-1



Problem Sketch and Finite Element Model

S6A, S6B: Deflection of a Cantilever Beam

TYPE:

Static analysis, plane stress element PLANE2D and SHELL6.

PROBLEM:

A cantilever beam is subjected to a concentrated load at the free end. Determine the deflections at the free end and the average shear stress.

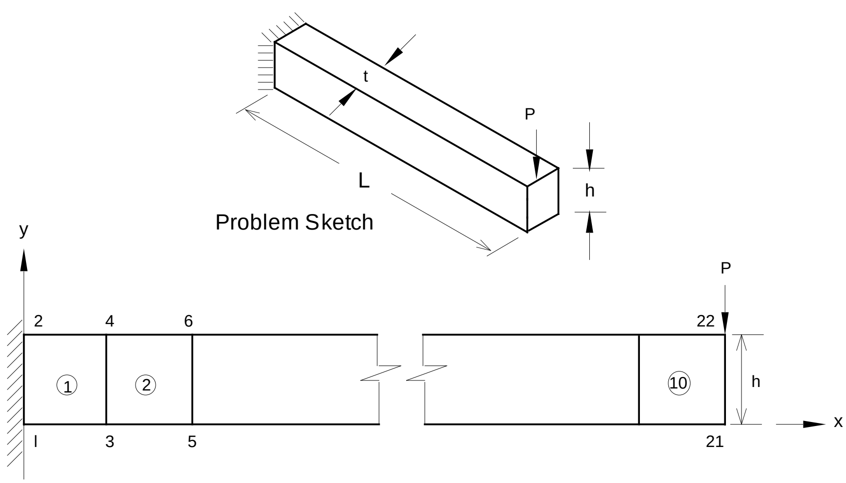
GIVEN:

E = 30 x 10^6 psi
L = 10 in
h = 1 in
A = 0.1 in^2
ν = 0
P = 1 lb
* averaged results of nodes at the free edge

COMPARISON OF RESULTS:

		Max. Deflection in the Y-direction	Shear Stress (psi)
Theory		-0.001333	-10.0
COSMOS/M	PLANE2D	-0.001337	-10.0*
	SHELL6 (Curved)	-0.0013398	-9.820667*
	SHELL6 (Assembled)	-0.00072411	--8.530667*

Figure S6-1



Finite Element Model

S7: Beam Stresses and Deflections

TYPE:

Static analysis, beam elements (BEAM3D).

REFERENCE:

Timoshenko, S. P., "Strength of Materials, Part 1, Elementary Theory and Problems," 3rd Ed., D. Van Nostrand Co., Inc., New York, 1965, p. 98.

PROBLEM:

A standard 30" Wide Flange beam is supported as shown below and loaded on the overhangs by a uniformly distributed load of 10,000 lb per ft. Determine the maximum stress in the middle portion of the beam and the deflection at the center of the beam.

GIVEN:

Area = $50.65\ in^2$

E = 30×10^6 psi

p = 10,000 lb/ft

COMPARISON OF RESULTS:

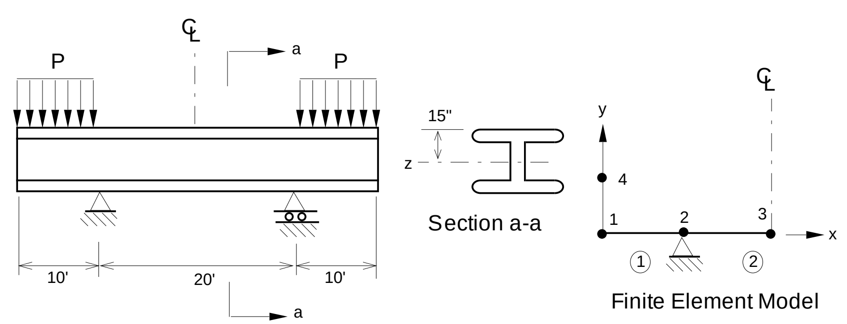
At the middle of the span (node 3):

	σ_{max} psi	δ inch
Theory	11400.0	0.182
COSMOS/M	11400.0	0.182

MODELING HINTS:

Use consistent length units. A half-model has been used because of symmetry. Resultant force and moment have been applied at node 2 instead of distributed load.

Figure S7-1



Problem Sketch

Finite Element Model

S8: Tip Displacements of a Circular Beam

TYPE:

Static analysis, thin or thick shell element (SHELL3).

REFERENCE

Warren C. Young, “Roark's Formulas for Stress and Strain,” Sixth Edition, McGraw Hill Book Company, New York, 1989.

PROBLEM:

Determine the deflections in X, Y direction of a circular beam fixed at one end and free at the other end, when subjected to a force along X direction at force end.

GIVEN:

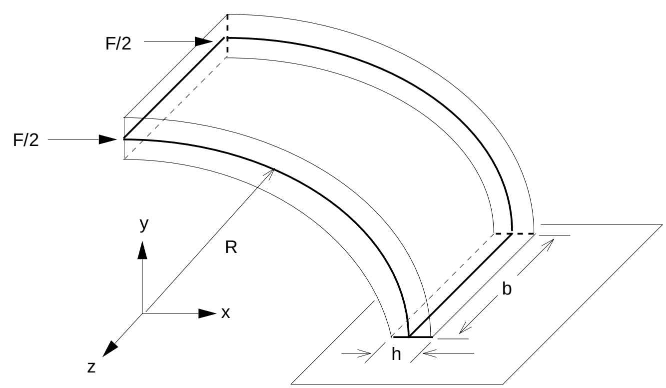
E = 30E6 psi
ν = 0
b = 4 in
h = 1 in
R = 10 in
F = 200 lb

COMPARISON OF RESULTS:

The loaded end.

	Displacement (inch)	
	X	Y
Theory	0.712E-2	0.99E-2
COSMOS/M	0.718E-2	0.99E-2

Figure S8-1



Problem Sketch and Finite Element Model

S9A: Clamped Beam Subject to Imposed Displacement

TYPE:

Static analysis, beam elements (BEAM2D).

REFERENCE

Gere, J. M. and Weaver, W. Jr., "Analysis of Framed Structures," D. Van Nostrand Co., 1965.

PROBLEM:

Determine the end forces of a clamped beam due to a 1 inch settlement at the right end.

GIVEN:

E = 30×10^6 psi

l = 80 in

A = 4 in^2

I = 1.33 in^4

h = 2 in

ANALYTICAL SOLUTION:

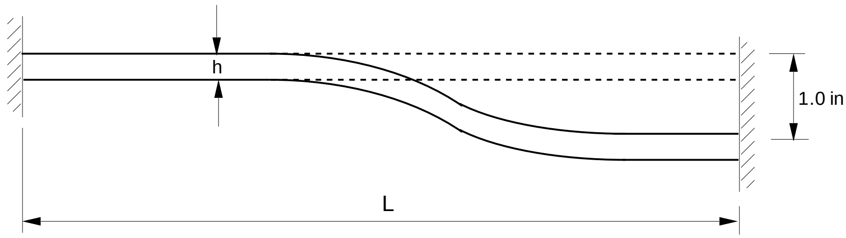
Reaction: $R = -12EI / L^3$

Moment: $M = 6EI / L^2$

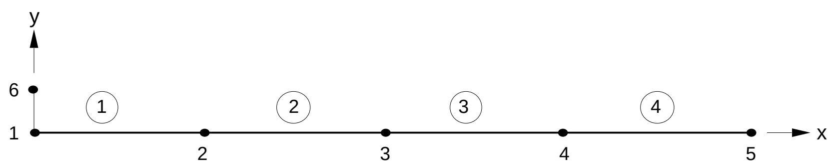
COMPARISON OF RESULTS:

	Theory	COSMOS/M
Imposed Displacement (in)	-1.0	-1.0
End Shear (lb)	-937.5	-937.5
End Moment (lb-in)	-37,500.0	-37,500.0

Figure S9A-1



Problem Sketch



Finite Element Model

S9B: Clamped Beam Subject to Imposed Rotation

TYPE:

Static analysis, beam elements (BEAM2D).

REFERENCE:

Gere, J. M. N. and Weaver, W. Jr., "Analysis of Framed Structures," D. Van Nostrand Co., 1965.

PROBLEM:

Determine the end forces of a clamped-clamped beam due to a 1 radian imposed rotation at the right end.

GIVEN:

E = 30×10^6 psi

l = 80 in

A = 4 in^2

I = 1.3333 in^4

h = 2 in

ANALYTICAL SOLUTION:

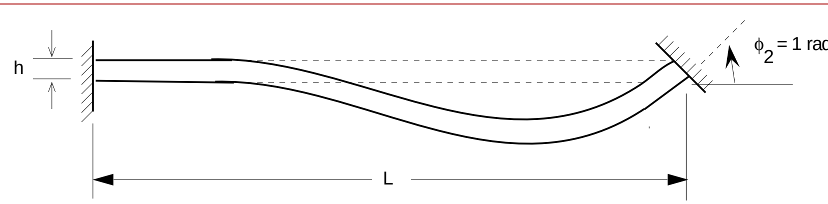
Reaction: $R = -6EI / L^2$

Moment: $M = 4EI / L$

COMPARISON OF RESULTS:

	Theory	COSMOS/M
Imposed Rotation (1 rad)	1	1
End Shear	-37,500	-37,500
End Moment	-2,000,000	-2,000,000

Figure S9B-1



Problem Sketch

S10A, S10B: Bending of a Solid Beam

TYPE:

Static analysis, SOLID element.

REFERENCE:

Roark, R. J., “Formulas for Stress and Strain,” 4th Edition, McGraw-Hill Book Co., New York, 1965, pp. 104-106.

PROBLEM:

A beam of length L and height h is built-in at one end and loaded at free end: (A) with a shear force F, and (B) a moment M. Determine the deflection at the free end.

GIVEN:

L = 10 in

h = 2 in

E = 30 x 10^6 psi

ν = 0

F = 300 lb

M = 2000 in-lb

MODELING HINTS:

Two load cases have been used (S10A, S10B).

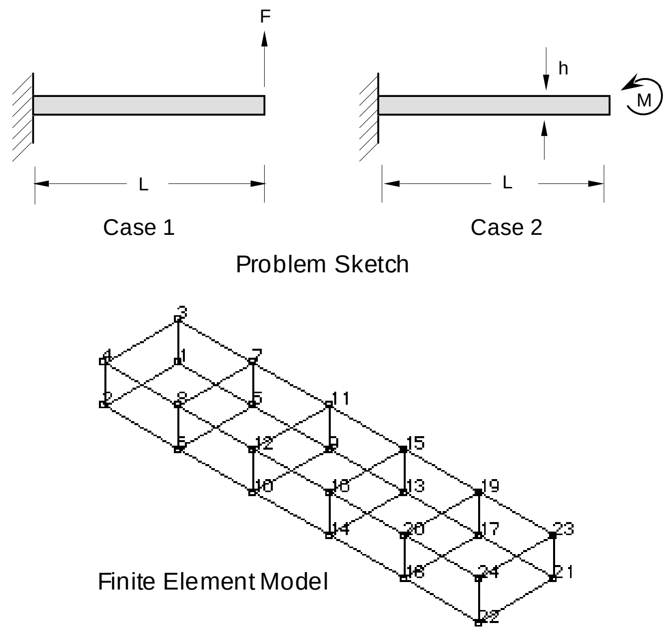
1. Four forces equal to F/4 have been applied at nodes 21, 22, 23, and 24 in xz direction (S10A), and,
2. Two couples equal M/2 have been applied at nodes 21, 22, 23 and 24 (S10B).

COMPARISON OF RESULTS:

Displacement in Z-direction (in) (node 21-24):

	S10A	S10B
Theory	0.00500	-0.00500
COSMOS/M	0.005007	-0.00495

Figure S10A-1



S11: Thermal Stress Analysis of a 3D Structure

TYPE:

Linear thermal stress analysis, 3D SOLID element.

PROBLEM:

Determine the displacements of the three-dimensional structure shown below due to a uniform temperature rise.

GIVEN:

E = 3×10^7 psi

α = 0.65×10^{-5}/° F

ν = 0.25

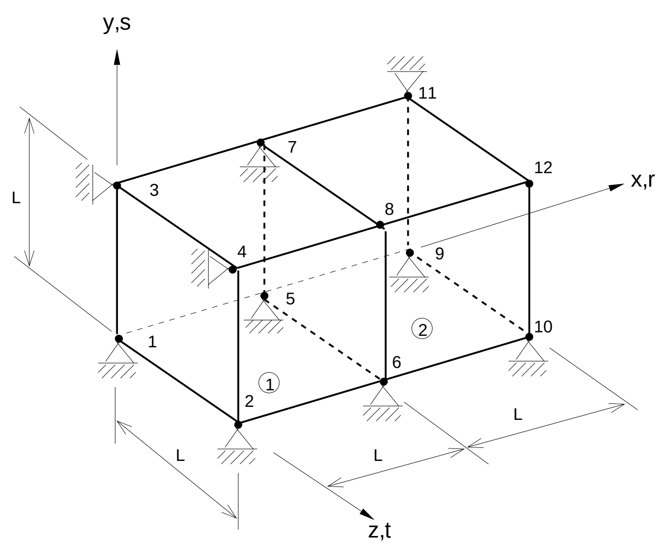
T = 100° F

L = 1 in

COMPARISON OF RESULTS:

	X-Displacement (Nodes)	
	5, 6, 7, 8	**9, 10, 11, 12**
Theory	0.000650	0.001300
COSMOS/M	0.000650	0.001300

Figure S11-1



Problem Sketch and Finite Element Model

S12: Deflection of a Hinged Support

TYPE:

Static analysis, truss element (TRUSS3D).

REFERENCE:

Timoshenko, S. P., and MacCullough, Glesson, H., “Elements of Strength of Materials,” D. Van Nostrand Co., Inc., 3rd edition, June 1949, p. 13.

PROBLEM:

A structure consisting of two equal steel bars, 15 feet long and with hinged ends, is submitted to the action of a vertical load P. Determine the forces in the members AB and BC along with the vertical deflection at B.

GIVEN:

P = 5000 lbs

θ = 30°

AB = BC = 15 ft

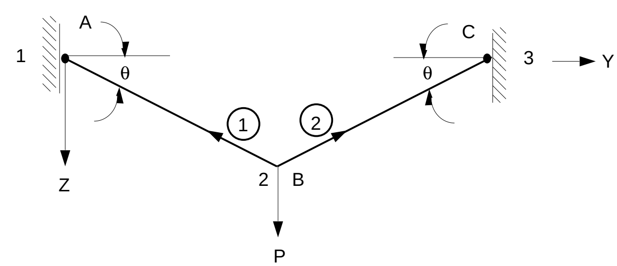
E = 30 x 10^6 psi

Cross-sectional area = 0.5 in^2

COMPARISON OF RESULTS:

	Theory	COSMOS/M
Vertical Deflection at B in inches	0.12	0.12
Forces in Members AB and BC in lbs	5000	5000

Figure S12-1



Problem Sketch and Finite Element Model

S13: Statically Indeterminate Reaction Force Analysis

TYPE:

Static analysis, truss elements (TRUSS3D).

REFERENCE:

Timoshenko, S. P., “Strength of Materials, Part 1, Elementary Theory and Problems,” 3rd edition, D. Van Nostrand Co., Inc., 1956, p. 26.

PROBLEM:

A prismatic bar with built-in ends is loaded axially at two intermediate cross-sections by forces F1 and F2. Determine the reaction forces R1 and R2.

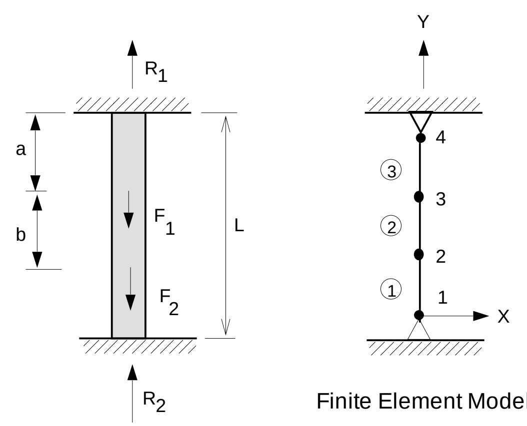
GIVEN:

a $= b = 0.3$ L
L $= 10$ in
F1 $= 2F_2 = 1000$ lb
E $= 30 \times 10^6$ psi

COMPARISON OF RESULTS:

	R_1 lbs	R_2 lbs
Theory	900	600
COSMOS/M	900	600

Figure S13-1



Problem Sketch

S14A, S14B: Space Truss with Vertical Load

TYPE:

Static analysis, truss elements (TRUSS3D).

REFERENCE:

Timoshenko, S. P. and Young, D. H. “Theory of Structures,” end Ed., McGraw-Hill, New York, 1965, pp. 330-331.

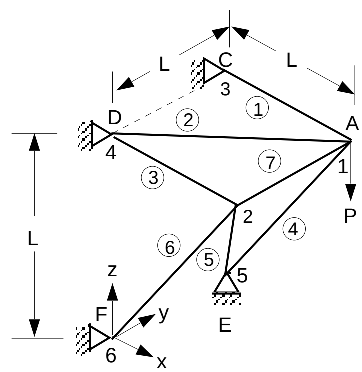
PROBLEM:

The simple space truss shown in the figure below consists of two panels ABCD and ABEF, attached to a vertical wall at points C, D, E, F, the panel ABCD being in a horizontal plane. All bars have the same cross-sectional area, A, and the same modulus of elasticity, E.

Calculate:

1. The axial force produced in the redundant bar AD by the vertical load P = 1 kip at joint A (S14A).
2. The thermal force induced in the bar AD if there is a uniform rise in temperature of 50° F (S14B).

Figure S14-1



Problem Sketch and Finite Element Model

GIVEN:

E = 30×10^6 psi

α = 6.5×10^{-6}/° F

A = 1 in^2

L = 4 ft

COMPARISON OF RESULTS:

For Element 2:

	S14A	S14B
Theory	56.0 lb	-1259.0 lb
COSMOS/M	55.92 lb	-1292.4 lb

S15: Out-of-Plane Bending of a Curved Bar

TYPE:

Static analysis, curved elbow element (ELBOW).

REFERENCE:

Timoshenko, S. P., "Strength of Materials, Part 1, Advanced Theory and Problems," 3rd Edition, D. Van Nostrand Company, Inc., New York, 1956, p. 412.

PROBLEM:

A portion of a horizontal circular ring, built-in at A, is loaded by a vertical load P applied at the end B. The ring has a solid circular cross-section of diameter d. Determine the deflection at end B, and the maximum bending stress.

GIVEN:

P = 50 lb
r = 100 in
d = 2 in
E = 30×10^6 psi
α = 90°
ν = 0.3

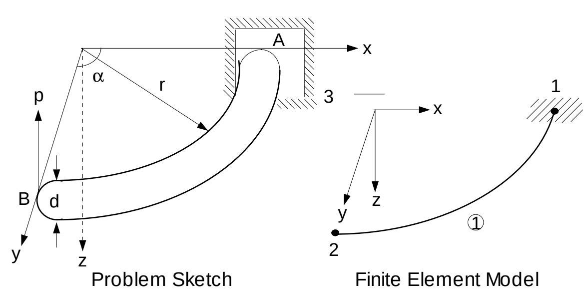
COMPARISON OF RESULTS

	δ_z, inch	σ_{Bend}, psi
Theory	-2.648	6366.0
COSMOS/M	-2.650	6366.2

MODELING HINTS:

COSMOS/M does not yet have a curved beam element, although this element will be incorporated into the program shortly. Hence, the curved elbow element is used to model this problem. Therefore, it is necessary to use equivalent thickness t which is equal to the radius of the solid rod.

Figure S15-1



S16A, S16B: Curved Pipe Deflection

TYPE:

Static analysis, elbow element (ELBOW).

REFERENCE:

Blake, A., "Design of Curved Members for Machines," Industrial Press, New York, 1966.

PROBLEM:

Calculate deflections x and y for a curved pipe shown in the figure subjected to:

1. Moment Mz = 3 x 10^6 lb-in and internal pressure p = 900 psi (S16A).
2. Internal pressure p = 900 psi (S16B).

GIVEN:

E = 30 x 10^6 psi

ν = 0.3

R = 72 in

Thickness = 1.031 in

Outer diameter of pipe = 20 in

COMPARISON OF RESULTS:

Blake gives the following results for a 90 curved member. These results do not include the effects of distortion of the cross-section and internal pressure.

$\delta y = M_z R^2/EI = 0.187039$ in

$\delta y = M_z R^2/EI\ (P/2-1) = 0.106761$ in

The pipe flexibility factor is given by

$Kp = 1.65/h\{1 + 6P/Eh)\ (R/t)^{4/3}\}$, where $h = tR/r^2$

for p = 900 psi, $K_p = 1.8814761$

To obtain the nodal deflections for case l, the deflections calculated by Blake's formulas must be multiplied by kp and added to the deflections produced by the internal pressure.

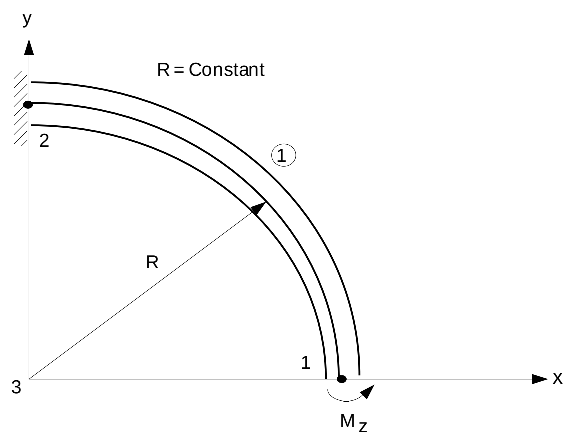
S16A

	δ_x, inch	δ_y, inch
Theory	0.37035	0.20515
COSMOS/M	0.37034	0.20515

S16B

	δ_x, inch	δ_y, inch
Theory	1.84356×10^{-2}	4.2873×10^{-3}
COSMOS/M	1.84355×10^{-2}	4.28043×10^{-3}

Figure S16A-1



Problem Sketch and Finite Element Model

S17: Rectangular Plate Under Triangular Thermal Loading

TYPE:

Linear thermal stress analysis, 2D elements (plane stress analysis, PLANE2D).

REFERENCE:

Johns, D. J., "Thermal Stress Analysis," Pergamon Press, Inc., 1965, pp. 40-47.

PROBLEM:

A finite rectangular plate is subjected to a temperature distribution in only one direction as shown in figure. Determine the normal stress at point A.

GIVEN:

a = 15 in

b = 10 in

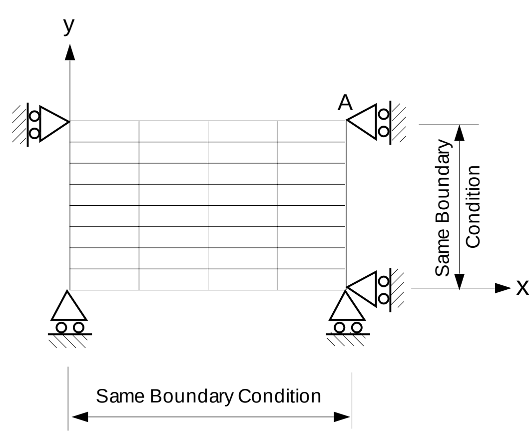
T_o = -100 °F

t = 1 in

E = 30×10^6 psi

α_c = 0.65×10^{-5} in/in/° F

Figure S17-1



MODELING HINTS:

Due to the double symmetry in geometry and loading, only one quarter of the plate was analyzed.

COMPARISON OF RESULTS:

		$\sigma_{xx} / (E \alpha T_o)$ (Node 45)
Reference	Method 1	0.42
	Method 2	0.40
COSMOS/M		0.437

S18: Hemispherical Dome Under Unit Moment Around Free Edge

TYPE:

Static linear analysis, axisymmetric shell element (SHELLAX).

REFERENCE:

Zienkiewicz, O. C. "The Finite Element Method," Third edition, McGraw-Hill Book Co., New York, 1983, p. 362.

PROBLEM:

Determine the horizontal displacement of a hemispherical shell under uniform unit moment around the free edge.

GIVEN:

R = 100 in
r = 50 in
E = 1 x 10^7 psi
ν = 0.33
t = 1 in
M = 1 in lb

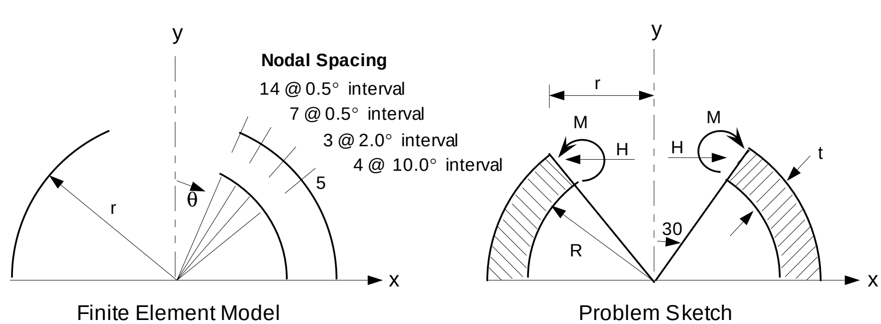
COMPARISON OF RESULTS:

	Horizontal Displacement (Node 29) (inch)
Reference	1.580 E-5
COSMOS/M	1.589 E-5

MODELING HINTS:

Nodal spacing is shown in the Figure. For convenience, cylindrical coordinate system is chosen for node generation. It is important to note that nodal load is to be specified per unit radian which in this case is 50 in lb/rad.

Figure S18-1



S19: Hollow Thick-walled Cylinder Subject to Temperature and Pressure

TYPE:

Static analysis, 2D axisymmetric element (PLANE2D).

REFERENCE:

Timoshenko, S. P. and Goodier, “Theory of Elasticity,” McGraw-Hill Book Co., New York, 1961, pp. 448-449.

PROBLEM:

The hollow cylinder in plane strain is subjected to two independent load conditions.

1. An internal pressure.
2. A steady state axisymmetric temperature distribution given by the equation:

   $T(r) = (Ta/\ln(b/a)) \cdot \ln(b/r)$ where Ta is the temperature of the inner surface and T(r) is the temperature at any radius.

GIVEN:

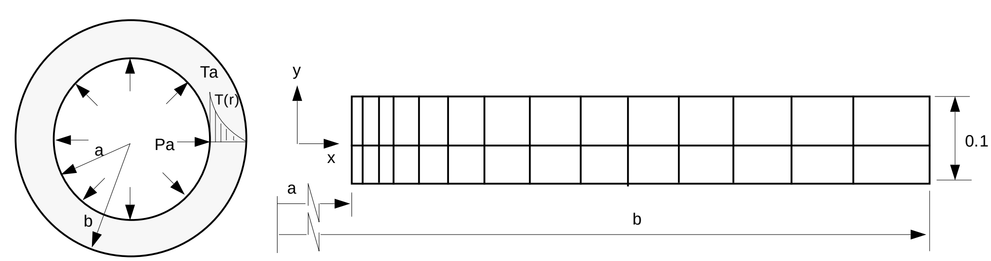
E = 30×10^{6} psi
a = 1 in
b = 2 in
ν = 0.3
α = 1×10^{-6} in/(in °F)
Pa = 100 psi
Ta = 100 °F

COMPARISON OF RESULTS:

At r = 1.2875 in (elements 13, 15)

	σ_r, psi	σ_θ, psi
Theory)	-398.34	-592.47
COSMOS/M	-398.15	-596.46

Figure S19-1



Problem Sketch Finite Element Model

S20A, S20B: Cylindrical Shell Roof

TYPE:

Static analysis, shell element (SHELL4, SHELL6).

REFERENCE:

Pawsley, S. F., “The Analysis of Moderately Thick to Thin Shells by the Finite Element Method,” Report No. USCEM 70-12, Dept. of Civil Engineering, University of California, 1970.

PROBLEM:

Determine the vertical deflections across the midspan of a shell roof under its own weight. Dimensions and boundary conditions are shown in the figure below.

GIVEN:

r	= 25 ft
E	= 3 x 10^6 psi
ν	= 0
Shell Weight	= 90 lbs/sq ft

MODELING HINTS:

Due to symmetry, a quarter of the shell is considered for modeling. The distributed force (self weight) is lumped at the nodes.

COMPARISON OF RESULTS:

Vertical Deflection at Midspan of free edge (Node 25):

		δ_x, (inch)
Theory		-0.3024
COSMOS/M	**SHELL4**	-0.3036
	SHELL6 (Curved)	-0.24580
	SHELL6 (Assembled)	-0.29353

Figure S20-1

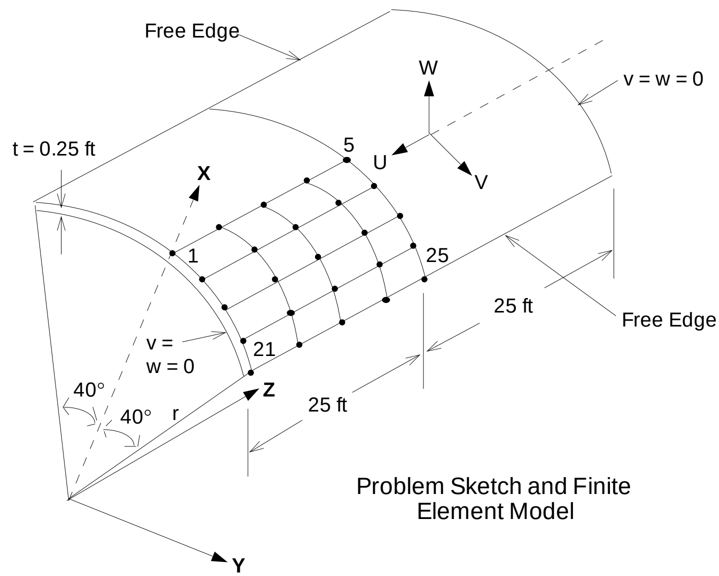
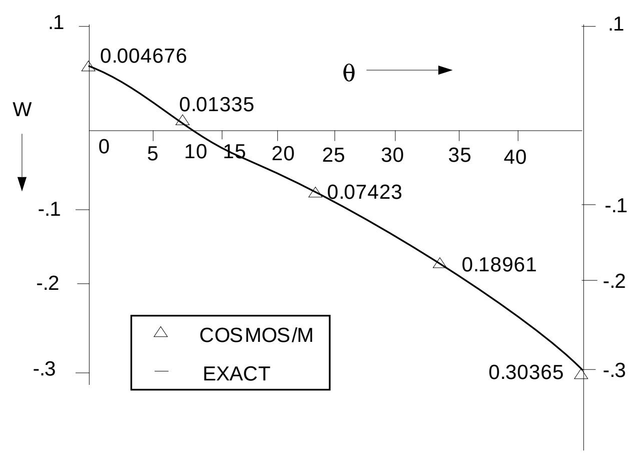


Figure S20-2



S21A, S21B: Antisymmetric Cross-Ply Laminated Plate (SHELL4L)

TYPE:

Static analysis, composite shell element (SHELL4L, SHELL9L).

REFERENCE:

Jones, Robert M., “Mechanics of Composite Materials,” McGraw-Hill, New York, 1975, p. 256.

PROBLEM:

Calculate the maximum deflection of a simply supported antisymmetric cross-ply laminated plate under sinusoidal load. The plate is made up of 6-layers and the material in each layer is orthotropic.

GIVEN:

a = 100 in

b = 20 in

h = 1 in

E_a = 40E6 psi

E_b = 1E6 psi

ν_{ab} = 0.25

$G_{ab} = G_{ac} = G_{bc}$ = 5E5 psi

For each layer, pressure loading = $\cos \pi x/a \cdot \cos \pi y/b$

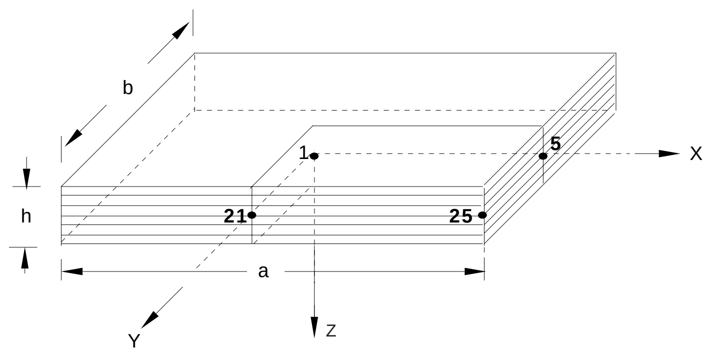
MODELING HINT:

Due to symmetry, a quarter of the plate is considered for modeling.

COMPARISON OF RESULTS:

		Maximum Deflection (in)
Theory		0.105E-2
COSMOS/M	4-nod shell	0.104E-2
	9-node shell	0.111E-2

Figure S21-1



Problem Sketch and Finite Element Model

S22: Thermally Loaded Support Structure

TYPE:

Static, thermal stress analysis, truss and beam elements (TRUSS3D, BEAM3D).

REFERENCE:

Timoshenko, S. P., "Strength of Materials, Part 1, Elementary Theory and Problems," 3rd Ed., D. Van Nostrand Co., Inc., 1956, p. 30.

PROBLEM:

Find the stresses in the copper and steel wire structure shown below. The structure is subjected to a load Q and a temperature rise of 10° F after assembly.

GIVEN:

Cross-sections area = 0.1 in^2

Q = 4000 lb

α_c = 92 x 10 in/in - °F

α_s = 70 x 10 in/in - °F

E_c = 16 x 10^6 psi

E_s = 30 x 10^6 psi

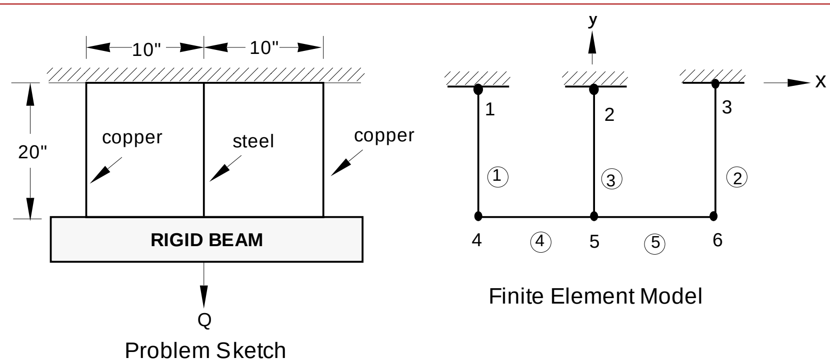
COMPARISON OF RESULTS:

	σ_{steel}, psi	σ_{copper}, psi
Theory	19695.0	10152.0
COSMOS/M	19704.2	10147.9

MODELING HINTS:

Length and spacing between wires are arbitrarily selected. Truss element is used for elements number (1), (2), and (3), and the beam element for elements (4) and (5). Beam type and material are arbitrarily selected.

Figure S22-1



S23: Thermal Stress Analysis of a Frame

TYPE:

Linear thermal stress analysis, beam elements (BEAM3D).

REFERENCE:

Rygol, J., "Structural Analysis by Direct Moment Distribution," Gordon and Breach Science Publishers, New York, l968, pp. 292-294.

PROBLEM:

An irregular frame subjected to differential temperature. Find member end moments.

Member Specifications				
Member	d (ft)	b (ft)	Ar-r (ft)	It-t (ft)
1	1.5	1.5	2.25	0.422
2	2.25	1.25	2.8125	1.187
3	2.0	1.5	3.0	1.0
4	2.5	1.25	3.125	1.628
5	2.0	1.5	3.0	1.0

GIVEN:

$E = 192857$ tons/ft^2

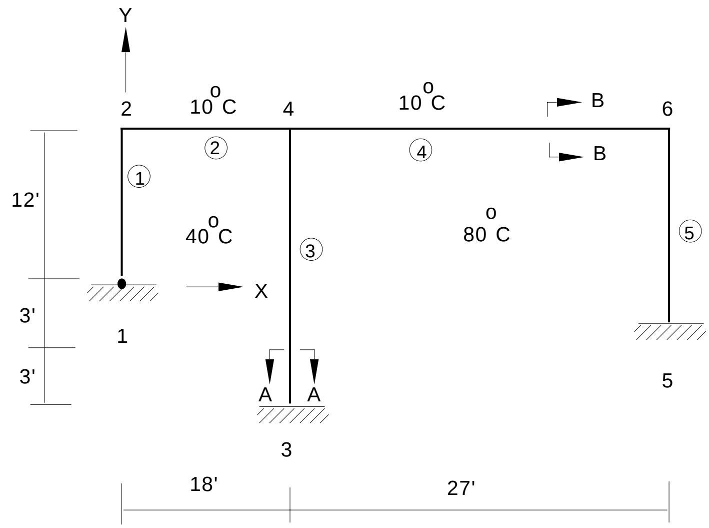
$\alpha = 0.00001$ ft/ft° C

COMPARISON OF RESULTS:

Moments (lb-in):

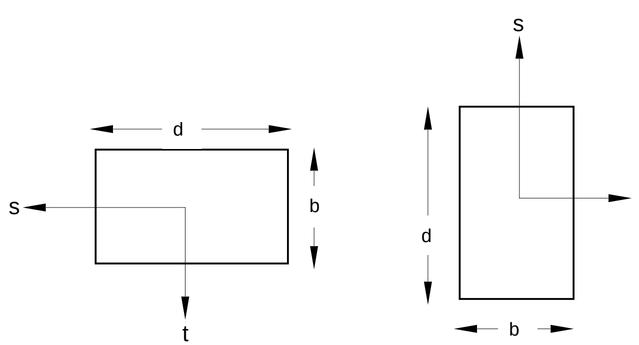
Member No.	COSMOS/M	Reference Solution
1	-17.96	-17.96
2	+17.96 -42.87	+17.96 -42.96
3	+38.73 -41.92	+38.64 -41.96
4	+84.79 -82.61	+84.92 -82.61
5	-57.50 +82.61	-57.40 +82.61

Figure S23-1



Problem Sketch and Finite Element Model

Figure S23-2



Section A-A Section B-B

S24: Thermal Stress Analysis of a Simple Frame

TYPE:

Linear thermal stress analysis, beam elements (BEAM2D).

PROBLEM:

Determine displacements and end forces of the frame shown in the figure below due to temperature rise at the nodes and thermal gradients of members as specified below.

GIVEN:

E = 30,000 kips/in^2

α = 0.65 x 10 in/in/° F

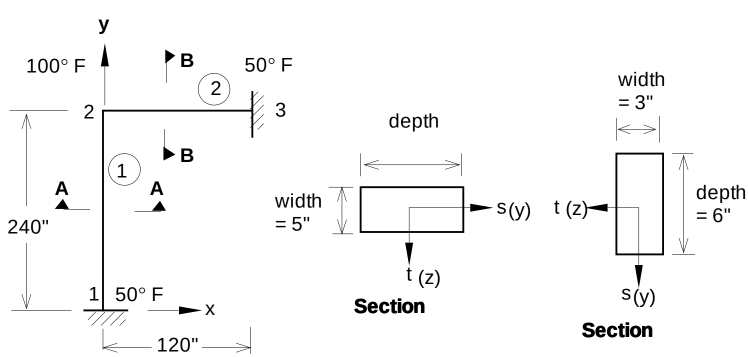
Element No.	Difference in Temperature (° F)	
	S-dir	T-dir
1	72	0
2	0	13.5

COMPARISON OF RESULTS:

Displacements at node 2 (in):

	δ_x	δ_y
Theory	-0.0583	0.1157
COSMOS/M	-0.0583	0.1168

Figure S24-1



Problem Sketch and Finite Element Model

S25: Torsion of a Square Box Beam

TYPE:

Static analysis, shell elements (SHELL4).

REFERENCE:

Timoshenko, S. P., and Goodier, J. N., “Theory of Elasticity,” McGraw-Hill, New York, 1951, p. 299.

PROBLEM:

Find the shear stress and the angle of twist for the square box beam subjected to a torsional moment T.

GIVEN:

E = 7.5 psi

ν = 0.3

t = 3 in

a = 150 in

L = 1500 in

T = 300 lb in

COMPARISON OF RESULTS:

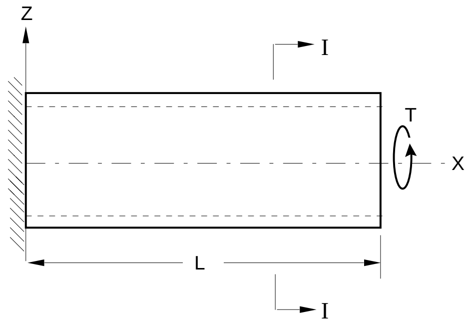
r

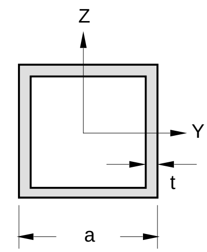
	Shear Stress τ psi	Rotation θ*, rad
Theory	0.00222	0.0154074
COSMOS/M	0.0021337 (average)	0.0154035*

* θ is calculated as:

θ = Sin^{-1}(resultant displacemet of node 25/distance from node 25 to the center of the cross section)

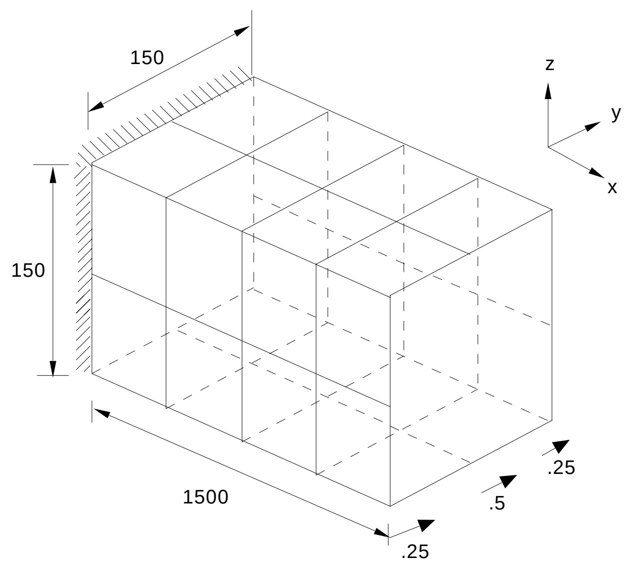
Figure S25-1





Section I-I

Problem Sketch



Finite Element Model

S26: Beam With Elastic Supports and a Hinge

TYPE:

Static analysis, beam and truss elements (TRUSS3D, BEAM3D).

REFERENCE:

Beaufait, F. W., et. al., “Computer Methods of Structural Analysis,” Prentice-Hall, Inc., New Jersey, 1970, pp. 197-210.

PROBLEM:

The final end actions of the members and the reactions of the supports resulting from the applied loading are to be determined for the structural system described in the figure below. At the beam-column connection, joint 3, the beam is continuous and the column is pin-connected to the beam.

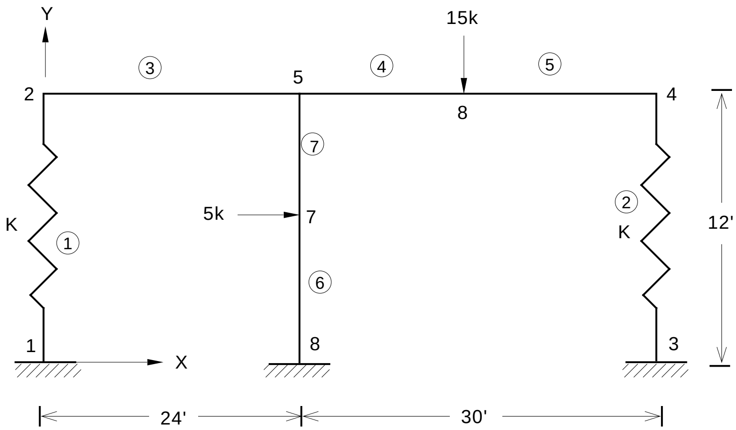
GIVEN:

Cross-sectional area of beams	$= A_1 = A_2 = 0.125\ ft^2$
Moment of inertia of beams	$= I_1 = I_2 = 0.263\ ft^4$
Cross-sectional area of column	$= A_3 = 0.175\ ft^2$
Moment of inertia of column	$= I_3 = 0.193$ ft4
E	$= 1.44 \times 10^4\ kip/ft^2$
K (spring stiffness)	= 1200 kips/ft

COMPARISON OF RESULTS:

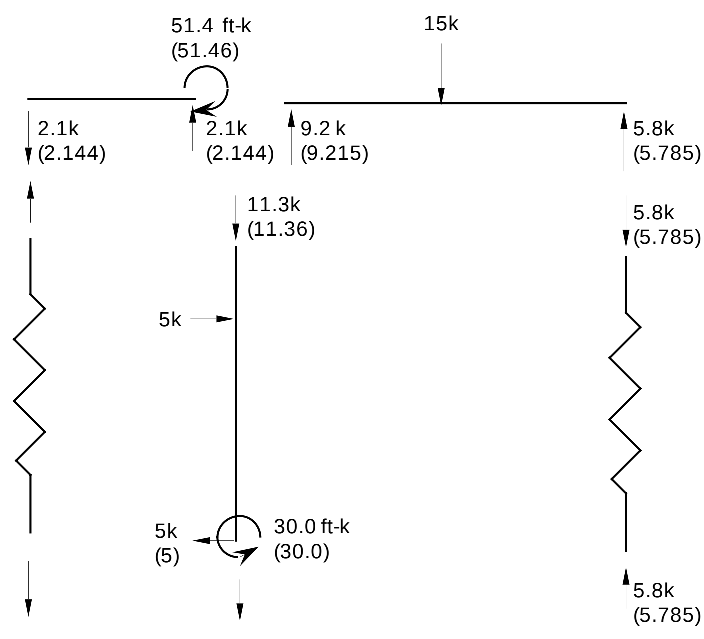
		Node 2	Node 4	Node 5
Reference	δ_x (10^{-3} ft)	1.0787	1.0787	1.0787
	δ_y (10^{-3} ft)	1.7873	-4.8205	-0.1803
	θ_z (10^{-3} rad)	0.0992	0.3615	-0.4443
COSMOS/M	δ_x (10^{-3} ft)	1.0794	1.0794	1.0794
	δ_y (10^{-3} ft)	1.7869	-4.8205	-0.1803
	θ_z (10^{-3} rad)	0.0992	0.3615	-0.4443

Figure S26-1



Problem Sketch and Finite Element Model

Figure S26-2



S27: Frame Analysis with Combined Loads

TYPE:

Static analysis, beam elements (BEAM3D).

REFERENCE:

Laursen, Harold I., “Structural Analysis,” McGraw-Hill Book Co., Inc., New York, 1969, pp. 310-312.

PROBLEM:

Determine the forces in the beam members under the loads shown in the figure. Consider two separate load cases represented by the uniform pressure and the concentrated force. Set up the input to solve each one individually and then combine them together to obtain the final result.

GIVEN:

I_{yy} = I_{zz} = 0.3215 ft^4

I = 0.6430 ft^4

A_1 = 3.50 ft^2

$A_{2,3}$ = 4.40 ft^3

A_4 = 2.79 ft^2

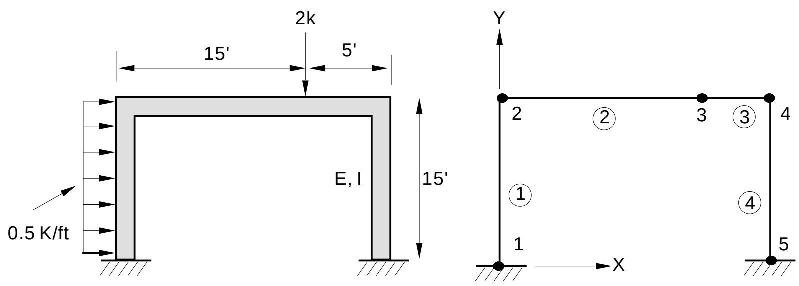
E = 432 x 10^4 K/ft^2

Areas of members were made to be larger than the actual area in order to neglect axial deformation.

COMPARISON OF RESULTS:

The results are shown in the figure below with COSMOS/M results shown in parentheses.

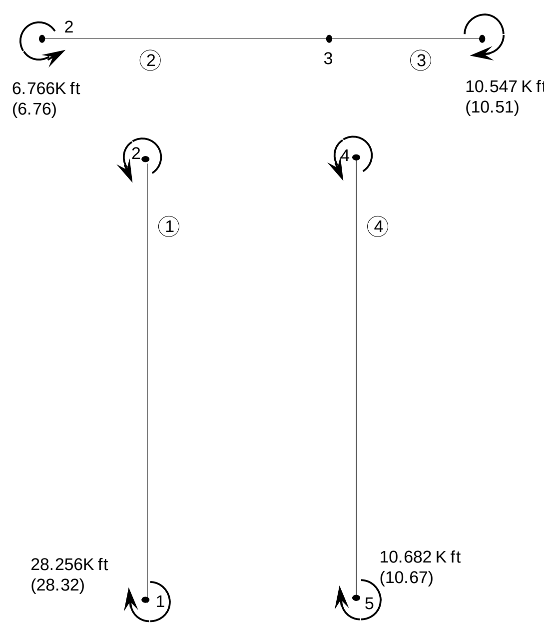
Figure S27-1



Problem Sketch

Finite Element Model

Figure S27-2



S28: Cantilever Unsymmetric Beam

TYPE:

Static analysis, 3D beam element (BEAM3D).

REFERENCE:

Boresi, A. P., Sidebottom, O. M., Seely, F. B., Smith, J. O., “Advanced Mechanics of Materials,” John Wiley and Son, Third Edition, 1978.

PROBLEM:

An unsymmetric cantilever beam is subjected to a concentrated load at the free end. Determine the tip displacement of the beam, the end forces and the stress at y = 8, z = -2 at the clamped end.

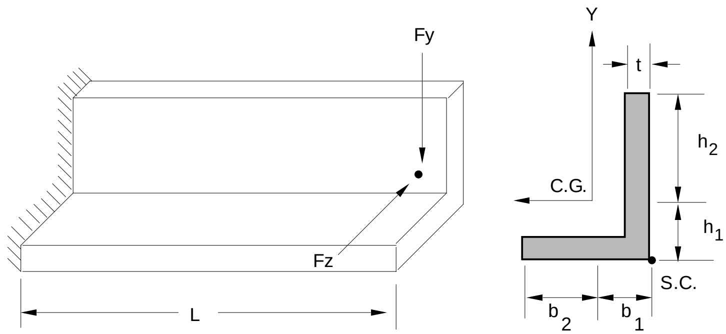
GIVEN:

E	$= 2 \times 10^7$ N/cm^2	b_2	= 6 cm
Fy	= -8 N	A	= 19 cm^2
h_1	= 4 cm	I_{yy}	= 100.3 cm^4
b_1	= 2 cm	I_{zz}	= 278.3 cm^4
L	= 500 cm	I_{yz}	= 97.3 cm^4
F_z	= -4 N	I_{xx}	= J = 6.333 cm^4
h_2	= 8 cm	t	= 1 cm

COMPARISON OF RESULTS:

	Theory	COSMOS/M
Node 6		
Translation in Y Dir (cm)	-0.1347	-0.1346
Translation in Z Dir (cm)	-0.2140	-0.21364
Rotation about X Axis (rad)	0	0
Node 1		
Moment about Y Axis (N-cm)	-2000.0	-2000.0
Shear in Z Dir (N)	4.0	4.0
Stress at Y = 8, Z = 2	155.74 Tension	155.8 Tension

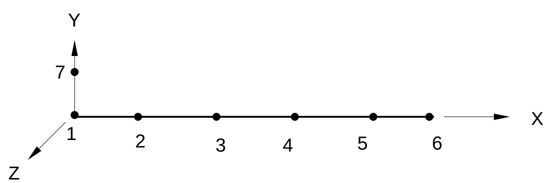
Figure S28-1



Unsymmetric Beam Structure

Cross Section

Problem Sketch



Finite Element Model

S29A, S29B: Square Angle-Ply Composite Plate Under Sinusoidal Loading

TYPE:

Static analysis, composite shell (SHELL9L), and solid element (SOLIDL).

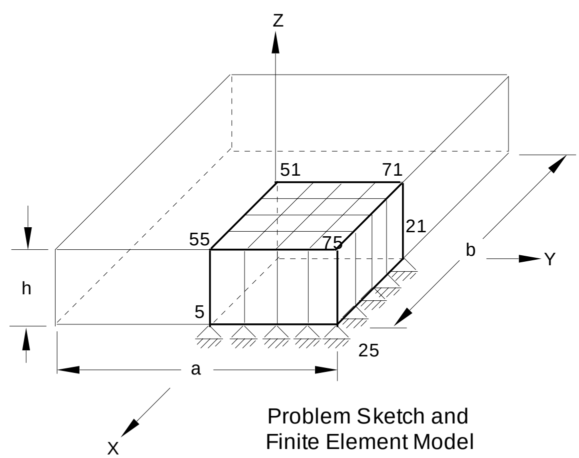
REFERENCE:

Jones, Robert M., “Mechanics of Composite Materials,” McGraw-Hill, N. Y., l975, p. 258.

PROBLEM:

Calculate the maximum deflection of a simply supported square antisymmetric angle-ply under SINUSOIDAL loading. The plate is made up of 6 layers, where the top layer material axis orientation makes 45 degree angle with x-axis. To impose simply-supported boundary conditions, 2 layers of composite solid elements (each has 3 layers of different material orientation) through the thickness are required.

Figure S29-1



Problem Sketch and Finite Element Model

GIVEN:

a = b = 20 in

E_a = 40E6 psi

h = 0.01 in

E_b = lE6 psi

ν = 0.25

$G_{ab} = G_{ac} = G_{bc}$ = 5E5 psi

p = cos (π x/a) cos (π y/b)

p_o = 1E-3

COMPARISON OF RESULTS:

		Max. Deflection
Reference Solution		0.256
COSMOS/M	SOLIDL	0.258
	SHELL9L	0.258

S30: Effect of Transverse Shear on Maximum Deflection

TYPE:

Static analysis, shell elements (SHELL3).

REFERENCE:

Pryor, Charles W., Jr., and Barker, R. M., “Finite Element Bending Analysis of Reissner Plates,” Engineering Mechanics Division, ASCE, EM6, December, l970, pp. 967-983.

PROBLEM:

Find the effect of transverse shear on maximum deflection of an isotropic simply supported plate subjected to a constant pressure, q.

GIVEN:

a = b = 24 in	H = varies according to thickness ratio (H/a)	ν = 0.3
E = 30E6 psi		q = 30 psi

MODELING HINTS:

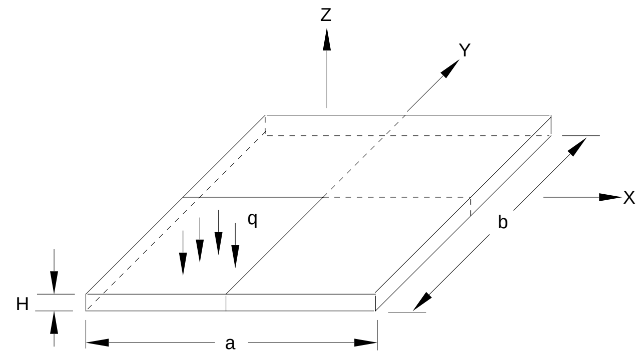
The input data corresponds to h = 0.1008 and the other inputs can be obtained by changing the thickness in the given input data. Due to symmetry, only one quarter of the plate is considered.

COMPARISON OF RESULTS:

Thickness Ratio H/a	Thickness H	β Coefficient*		Difference (%)
		Reissner Theory	COSMOS/M	
0.000042	0.001008	0.04436	0.04438**	0.045
0.00042	0.01008	0.04436	0.04438**	0.045
0.0042	0.1008	0.04436	0.04438**	0.045
0.05	1.20	0.044936	0.044772 ***	0.36
0.1	2.40	0.046659	0.046510 ***	0.32
0.15	3.60	0.049533	0.049405 ***	0.26
0.2	4.80	0.053555	0.053458 ***	0.180
0.25	6.00	0.058727	0.058669 ***	0.10
0.3	7.20	0.065048	0.065038 ***	0.02

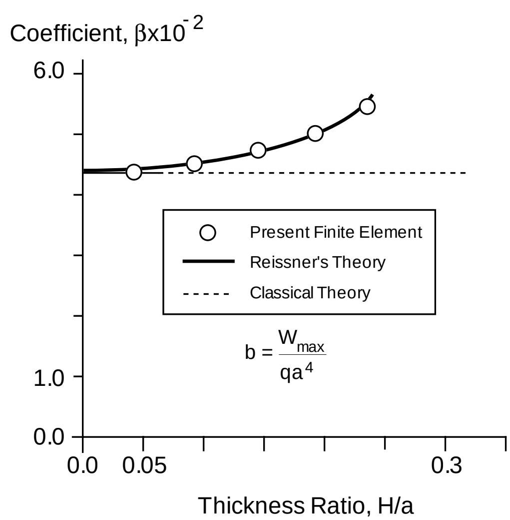
*$\beta = EH^3W_{max}/qa^4$ **Thin Shell (SHELL3) **Thick Shell (SHELL3T)

Figure S30-1



Problem Sketch

Figure S30-2



S31: Square Angle-Ply Composite Plate Under Sinusoidal Loading

TYPE:

Static analysis, shell element (SHELL4L).

REFERENCE:

Jones, Robert M., “Mechanics of Composite Materials,” McGraw Hill, N. Y., 1975, p. 258.

PROBLEM:

Calculate the maximum deflection of a simply supported square antisymmetric angle-ply under sinusoidal loading. The plate is made of 4-layers where the top layer material axis orientation makes 15 degree angle with the X-axis.

GIVEN:

$a = b = 20$ in

$h = 1$ in

$E_a = 40E6$ psi

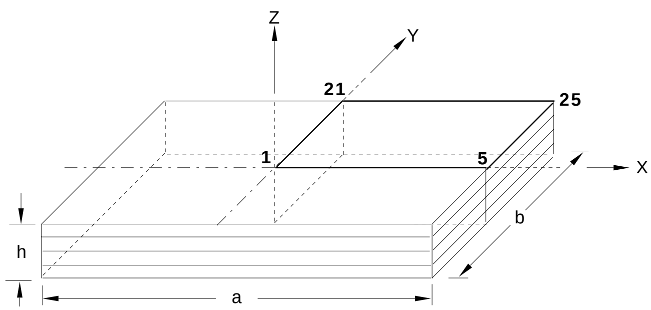
$E_b = 1E6$ psi

$\nu = 0.25$

$G_{ab} = G_{ac} = G_{bc} = 5E5$ psi

$p = \cos(\pi x/a) \cos(\pi y/b)$

Figure S31-1



Problem Sketch and Finite Element Model

COMPARISON OF RESULTS:

	W_{max} (inch)
Theory	4.24E-4
COSMOS/M	4.40E-4

S32A, S32B, S32C, S32D, S32M: Substructure of a Tower

TYPE:

Static analysis, substructuring using truss elements (TRUSS2D).

PROBLEM:

Determine the deflections of a tower loaded at top, using multi-level substructures.

GIVEN:

E = 10×10^6 psi
P = 1000 lb
h = 100 in
L = 30 in

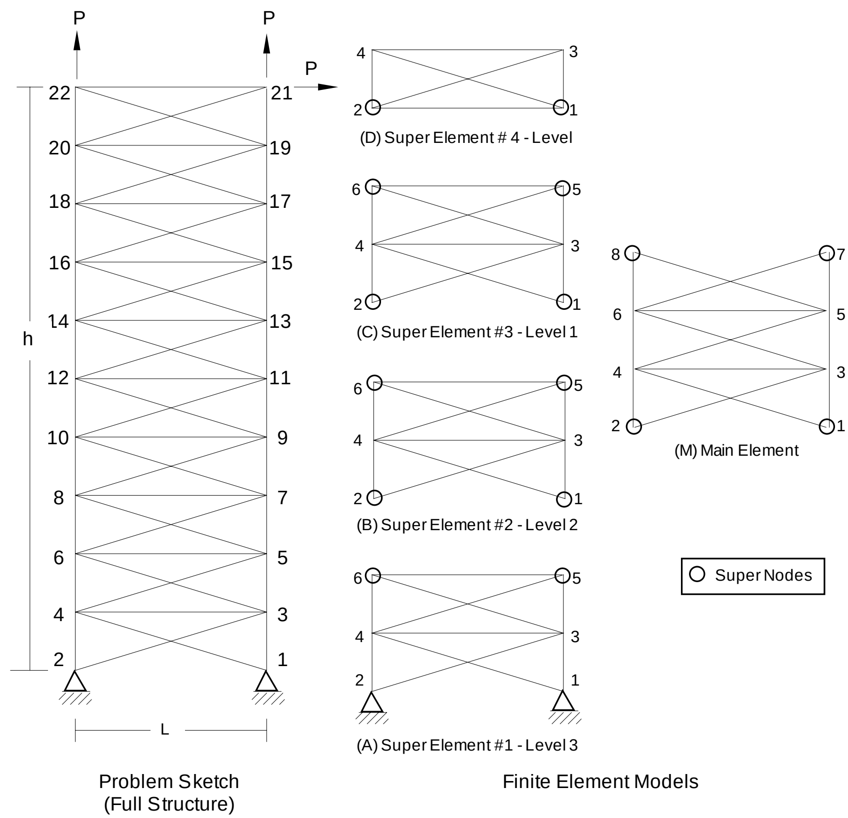
Cross-sectional areas of vertical and horizontal bars = 1 in^2

Cross-sectional areas of diagonal bars = 0.707 in^2

COMPARISON OF RESULTS:

Node Number		Deflection (10^{-3} inch)			
Full Structure	Sub-structure	COSMOS/M Using Full Structure		COSMOS/M Using Substructure	
		X	Y	X	Y
1	1(A)	0	0	0	0
2	5(A)	7.9761	9.1679	7.9761	9.1678
9	5(B)	14.6226	25.3632	14.6226	25.3631
13	5(C)	19.9360	46.8896	19.9359	46.8893
17	5(M)	23.9168	71.9909	23.9167	71.9905
21	3(D)	26.5878	99.3979	26.5877	99.3973
4	4(A)	-2.1815	3.4329	-2.1815	3.4328
6	2(B)	-4.0240	8.9940	-4.0239	8.9940
10	2(C)	-6.7108	25.1829	-6.7108	25.1828
14	2(M)	-8.0642	46.7075	-8.0641	46.7072
18	6(M)	-8.0834	71.7683	-8.0833	71.7678
22	4(D)	-6.7458	98.0790	-6.7457	98.0784

Figure S32A-1



S33A, S33M: Substructure of an Airplane (Wing)

TYPE:

Static analysis, substructuring using shell, beam and truss elements (SHELL4, BEAM3D, TRUSS3D).

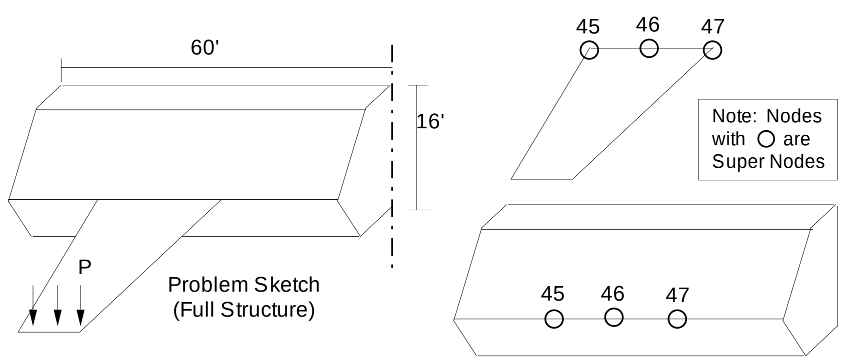
PROBLEM:

By using substructure method, determine the deflection of an airplane through the assembly of the calculations concerning separate parts, of the plane.

COMPARISON OF RESULTS:

Node Number	Deflection (Z-Direction) (inch) COSMOS/M Using Full Structure	Deflection (Z-Direction) (inch) COSMOS/M Using Substructure
16	10.100	10.178
17	8.0671	8.1024
18	13.800	13.894
19	10.666	10.708
20	8.7693	8.9513
21	12.276	12.958

Figure S33A-1



S34: Tie Rod with Lateral Loading

TYPE:

Static analysis, stress stiffening, beam elements (BEAM3D).

REFERENCE:

Timoshenko, S. P., "Strength of Materials, Part II, Advanced Theory and Problems," 3rd Edition, D. Von Nostrand Co., Inc., New York, 1956, p.42.

PROBLEM:

A tie rod subjected to the action of a tensile force S and a uniform lateral load q. Determine the maximum deflection z, and the slope at the left end. In addition, determine the same two quantities for the unstiffened tie rod (S = 0).

GIVEN:

L = 200 in

E = 30E6 psi

S = 21,972.6 lb

q = 1.79253 lb/in

b = h = 2.5 in

CALCULATED INPUT:

Area = 6.25 in^2

I = 3.2552 in^4

MODELING HINTS:

Due to symmetry, only one-half of the beam is modeled.

COMPARISON OF RESULTS:

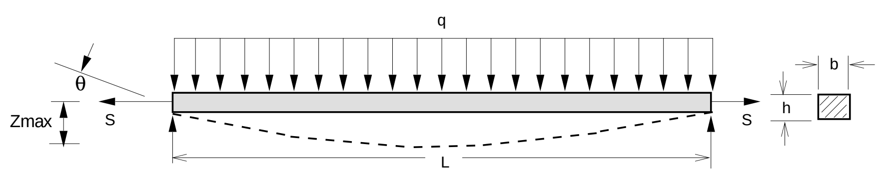
S ≠ 0 (Stiffened)

	Z_{max}, in	θ rad
Theory	-0.2	0.0032352
COSMOS/M	-0.19701	0.0031776

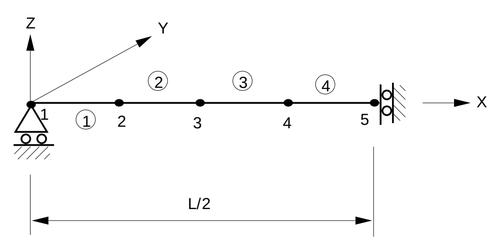
S = 0 (Unstiffened)

	Z_{max}, in	θ rad
Theory	-0.382406	0.006115
COSMOS/M	-0.37763	0.0060229

Figure S34-1



Problem Sketch



Finite Element Model

S35A, S35B: Spherical Cap Under Uniform Pressure (Solid)

TYPE:

Static analysis, solid and composite solid elements (SOLID, SOLIDL).

REFERENCE:

Reddy, N. J. "Exact Solutions of Moderately Thick Laminated Shells," J. Eng. Mech. Div. ASCE, Vol. 110, (1984), pp. 794-809.

PROBLEM:

Calculate the center deflection of a simply supported spherical cap under uniform pressure (q = 1.) in the direction normal to the cap surface. To impose simply-supported boundary conditions by solid elements, 2 layers of elements through the thickness are required.

Two types of material properties are being tested, each by a different solid element.

A. Isotropic material is handled by SOLID element (S35A).

B. Composite material, 4 layers with the orientation $0°/90°/90°/0°$, is analyzed by SOLIDL element (S35B). The lower layer of element is modeled by 2 layers of material orientation $0°/90°$ and the upper one is by $90°/0°$.

To capture the geometry of a curved surface by a bi-linear shape function accurately, at least 8 elements per side have to be used. The model used below is an 8x8x2 mesh.

GIVEN:

Geometry:

R = 96

h = 0.32 in

Length of side a =b = c = d = 32 in

Material Properties:

1. S35A: Isotropic
   - E = 1E7 psi
   - ν = 0.3
2. S35B: Composite $0°/90°/90°/0°$
   - E_x = 25E6 psi
   - $E_y = E_z$ = 1E6 psi
   - ν_{xy} = 0.25
   - $\nu_{yz} = \nu_{xy} = 0$
   - G_{yz} = 0.2E6 psi
   - $G_{xy} = G_{xz}$ = 0.5E6 psi

MODELING HINTS:

Boundary Conditions

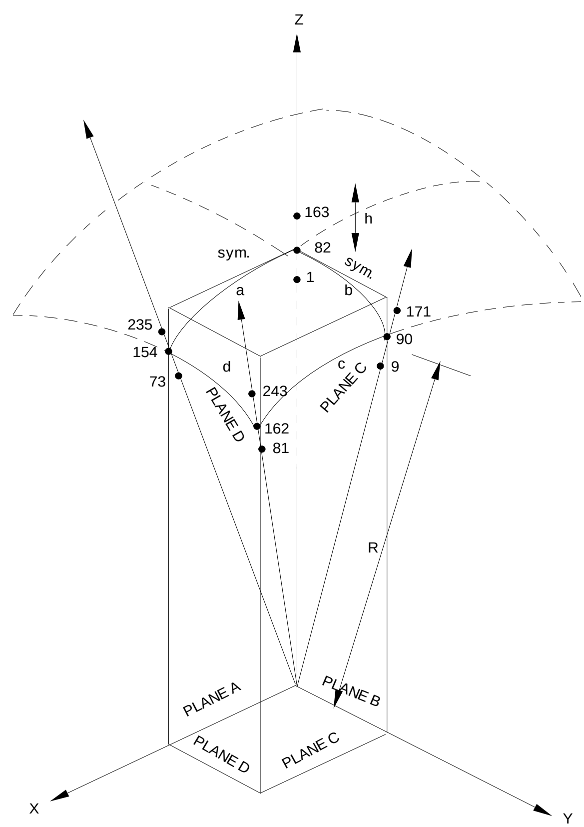
Due to symmetry:

1. All nodes on plane A, Uy = 0
2. All nodes on plane B, Ux = 0

Simply supported:

1. All nodes on side C, radial displacement = 0, Disp. on plane C = 0
2. All nodes on side D, radial displacement = 0, Disp. on plane D = 0

Figure S35A-1



Problem Sketch and Finite Element Model

S36A, S36M: Substructure of a Simply Supported Plate

TYPE:

Static analysis, substructuring using plate elements (SHELL4).

REFERENCE:

Timoshenko, S., “Theory of Plates and Shells,” McGraw-Hill, New York, 1940, p. 113-198.

PROBLEM:

Calculate the deflections of a simply supported isotropic plate subjected to uniform pressure p using the substructuring technique. Nodes 11 through 15 are super nodes which connect the substructure to the main structure.

GIVEN:

E = 30 x 10 psi

ν = 0.3

h = 0.5 in

p = 5 psi

a = 16 in

b = 10 in

MODELING HINTS:

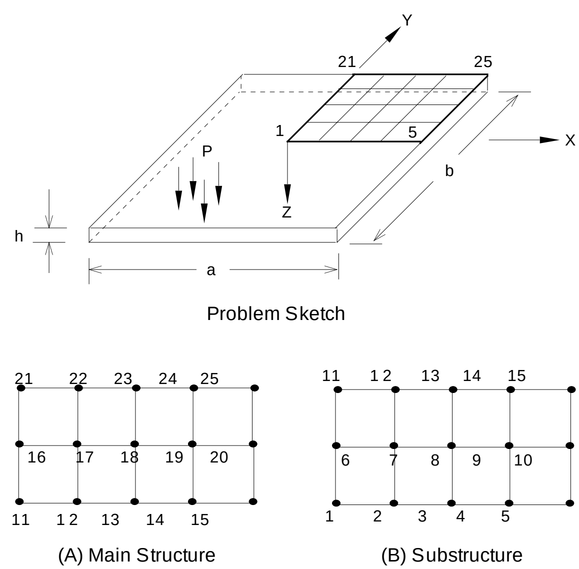
Due to symmetry, a quarter of a plate is taken for modeling.

COMPARISON OF RESULTS:

Timoshenko gives the expression for deflection w in the z-direction with origin at the corner of the plate.

Node No.	X (inch)	Y (inch)	W (inch)	
			Theory	COSMOS/M
1	0	0	0.0012103	0.00121329
2	2	0	0.0011338	0.00113636
3	4	0	0.0009043	0.00090247
4	6	0	0.0005150	0.00051044

Figure S36A-1



Finite Element Models

S37: Hyperboloidal Shell Under Uniform Ring Load Around Free Edge

TYPE:

Static analysis, axisymmetric shell elements (SHELLAX).

REFERENCE:

William Weaver, Jr., and Paul R. Johnston, "Finite Elements for Structural Analysis," Prentice-Hall, Inc., 1984, p. 275.

PROBLEM:

Determine the horizontal displacement of a hyperboloidal shell under uniform ring load around free edge.

GIVEN:

R_0	= 600 in	t	= 8 in
R_1	= 1200 in	E	= 3000 kip/sq in
H	= 2400 in	ν	= 0.3
H_0	= 1500 in	P	= 1 kip/in

Equation of the hyperboloid:

$$X^2 = 0.48\ (Y - H_0)^2 + R^2$$

MODELING HINTS:

Nodes at the top of the tower are spaced closely because of the concentrated ring load. Nodal spacing is as follows:

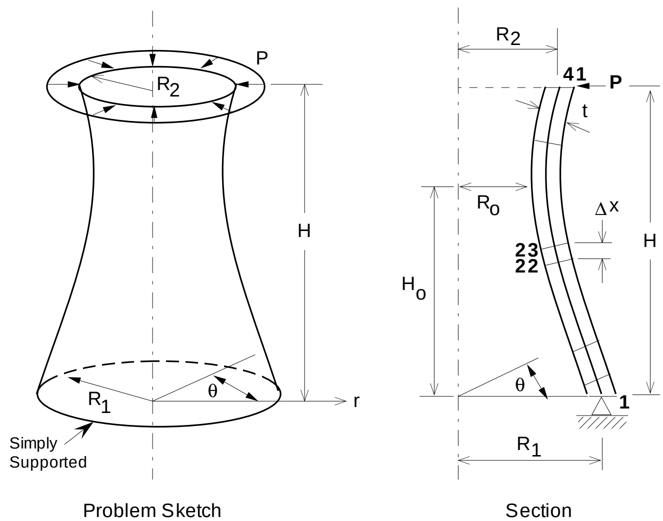
Nodes	1-11	11-21	21-29	29-39
D_y (in)	10	20	75	150

And it is to be noted that the ring load should be input per radian length, since the radius at the top of the shell is 865.3323 in, the load is 865.33 kip/rad.

COMPARISON OF RESULTS:

	Maximum Displacement at Node 41 (inch)
Theory	-0.904
COSMOS/M	-0.89705

Figure S37-1



S38: Rotating Solid Disk

TYPE:

Static analysis, axisymmetric (PLANE2D) elements, centrifugal loading.

REFERENCE:

S. P. Timoshenko and J. N. Goodier, “Theory of Elasticity,” McGraw-Hill, New York, 1970, p. 80.

PROBLEM:

A solid disk rotates about center 0 with angular velocity ω Determine the stress distribution in the disk.

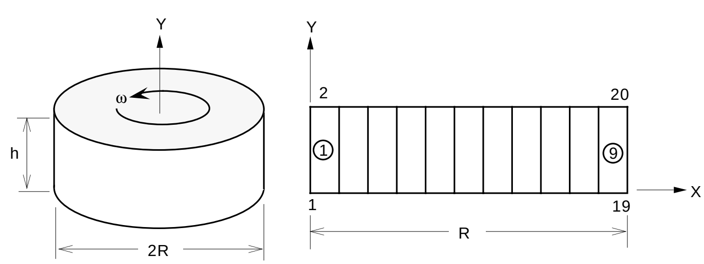
GIVEN:

E	$= 30 \times 10^6$ psi	h	= 1 in
DENS	$= 0.02$ lb sec^2/in^4	ω	= 25 rad/sec
ν	= 0.3	R	= 9 in

COMPARISON OF RESULTS:

	Location Element 1 (r = 0.5 inch)		Location Element 9 (r = 8.5 inch)	
	σ_r psi	σ_θ psi	σ_r psi	σ_θ psi
Theory	416.37	416.91	45.12	203.16
COSMOS/M	416.82	416.82	46.18	202.03

Figure S38-1



Problem Sketch and Finite Element Model

S39: Unbalanced Rotating Flywheel

TYPE:

Static analysis, centrifugal loading, beam and mass elements (BEAM3D, MASS).

PROBLEM:

The model shown in the figure is assumed to be rotating about the y-axis at a constant angular velocity of 25 rad/sec. Determine the axial forces and bending moments in the supporting beams and columns due to self-weight and rotational inertia.

GIVEN:

Figure S39-1

Elen

A :

Iyy :

Ixx :

E :

G :

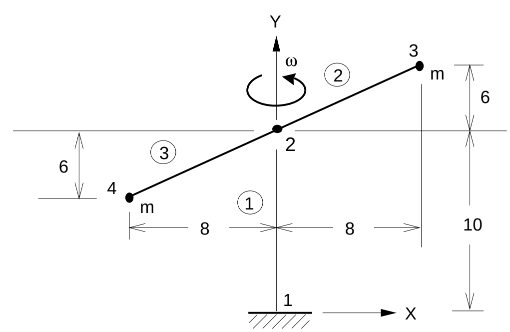
ρ :

Inerti

m_3 :

a_y :

w :



Problem Sketch and Finite Element Model

COMPARISON OF RESULTS:

	Theory	COSMOS/M
Element 2, Node 2		
Axial Force	58,800	58,800
Bending Moment	412,000	412,000
Element 3, Node 2		
Axial Force	61,200	61,200
Bending Moment	388,000	388,000

S40: Truss Structure Subject to a Concentrated Load

TYPE:

Static analysis, truss elements (TRUSS2D).

REFERENCE:

Hsieh, Y. Y., “Elementary Theory of Structures,” Prentice-Hall Inc., 1970, pp. 162-163.

PROBLEM:

Calculate the reactions and the vertical deflection of joint 2 of the loaded truss shown below subject to a concentrated load.

GIVEN:

E = 30,000 kips/in^2

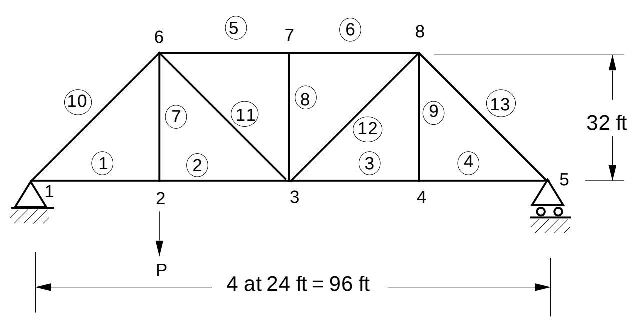
P = 64 kips

L (ft)/A(in) = 1 for all members

COMPARISON OF RESULTS:

	Theory	COSMOS/M
Deflection of Joint 2	0.006733 in	0.006733 in
Reaction at Node 1	48 K	48 K
Reaction at Node 5	16 K	16 K

Figure S40-1



Problem Sketch and Finite Element Model

S41: Reactions of a Frame Structure

TYPE:

Static analysis, beam element (BEAM2D).

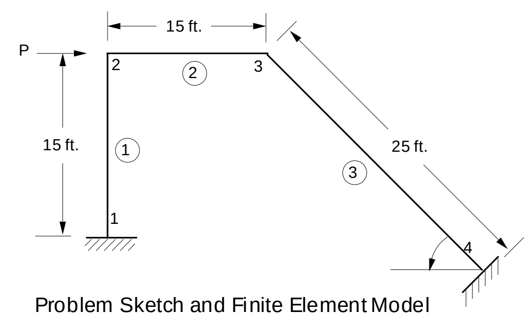
REFERENCE:

Hsieh, Y. Y., "Elementary Theory of Structures," Prentice-Hall Inc., 1970, pp. 258-259.

PROBLEM:

Determine the reactions for the frame shown below.

Figure S41-1



Problem Sketch and Finite Element Model

GIVEN:

E = 30×10^6 psi

A = 0.1 in^2

The relative values of 2EI/L:

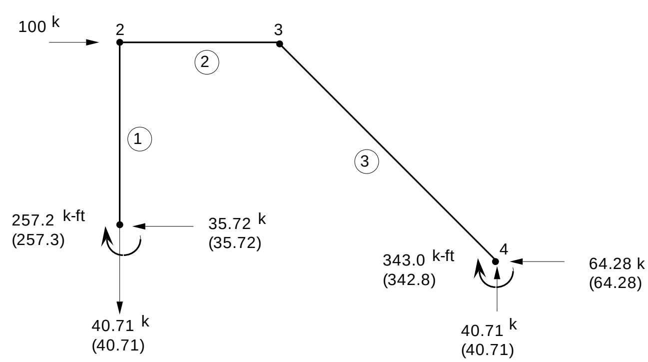
for element 1 = 1 lb-in

for elements 2, 3 = 2 lb-in

COMPARISON OF RESULTS:

The free body diagram for the structural system is given below and COSMOS/M results are given in parentheses.

Figure S41-2



S42A, S42B: Reactions and Deflections of a Cantilever Beam

TYPE:

Static analysis, shell elements (SHELL4, SHELL6).

PROBLEM:

Calculate reactions and deflections of a cantilever beam subject to a concentrated load at tip.

GIVEN:

E = 30E6 psi

h = 1 in

L = 10 in

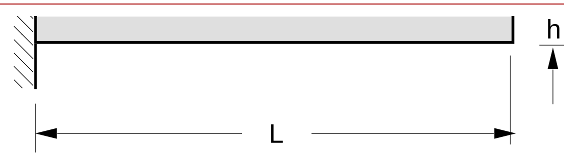
W = 4 in

P = 8 lb

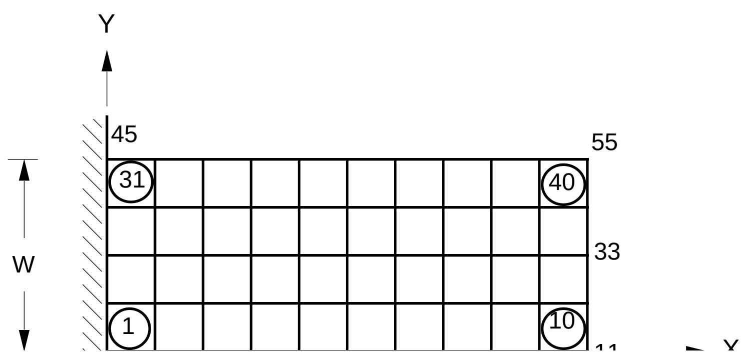
COMPARISON OF RESULTS:

	Theory	COSMOS/M		
		SHELL4	SHELL6 (Curved)	SHELL6 (Assembled)
Tip Deflection (Node 33)	-2.667×10^{-4}	-2.667×10^{-4}	-2.683×10^{-4}	-2.667×10^{-4}
Total Force Reaction	8 lb	8 lb	8 lb	8 lb
Total Moment Reaction	-80 lb-in	-80 lb-in	-80 lb-in	-80 lb-in

Figure S42-1



Problem Sketch



S43: Bending of a T Section Beam

TYPE:

Static analysis, shell element, beam element with offset (SHELL4L, BEAM3D).

PROBLEM:

Calculate the deflections and stresses of a cantilever T beam subjected to a concentrated load at the free end.

GIVEN:

L = 2000 in

y = 49 in

I = 480833.33 in^4

E = 10E10 psi

Dy = -24 in

COMPARISON OF RESULTS:

	Theory	COSMOS/M
Free End (at Node 12)		
Y-Displacement (in)	-5.546E-6	-5.588E-6
θ_z- Rotation	-4.159E-9	-4.161E-9
Clamped End		
σ_x top (psi)	4.57360	4.377
σ_x bottom (psi)	20.3813	21.302

ANALYTICAL SOLUTION:

$\delta = PL^3 / 3EI$

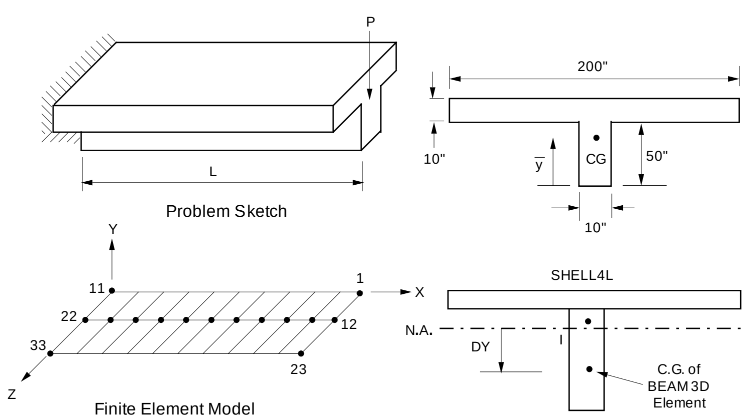
$\phi = PL^2 / 2EI$

$\sigma = Mc / I$

NOTE:

The maximum stress occurs in the beam. The point at which stresses are calculated for unsymmetric beams should be specified in the real constant set (real constants 25 and 26).

Figure S43-1



S44A, S44B: Bending of a Circular Plate with a Center Hole

TYPE:

Static analysis, shell elements (SHELL4), coupled points (file S44A) and/or constraint equations (file S44B).

REFERENCE:

Timoshenko, S., “Strength of Materials, Part II, Advanced Theory and Problems,” 3rd Edition, D. Van Nostrand Co., Inc., New York, l956.

PROBLEM:

A circular plate with a center hole is built-in along the inner edge and unsupported along the outer edge. The plate is subjected to bending by a moment M applied along the outer edge. Determine the maximum deflection and the maximum slope of the plate. In addition, deter-mine the moment M and the corresponding stress at the center of the first and the last elements.

GIVEN:

E	= 30E6 psi	a	= 30 in
ν	= 0.3	M	= 10 in lb/in
h	= 0.25 in	θ	= 10°
b	= 10 in		

CALCULATED INPUT:

M_{1a} = 10 in-lb/in = 52.359 in lb/l0° segment

MODELING HINTS:

Since the problem is axisymmetric, only a small sector of elements is needed. A small angle θ is used for approximating the circular boundary with a straight-side element. A radial grid with nonuniform spacing (3:l) is used. The load is applied equally to the outer nodes. Coupled nodes (CPDOF) and/or constraint equations (CEQN) are used to ensure symmetry for S44A and S44B, respectively. Note that all constraint and load commands are active in the cylindrical coordinate system.

COMPARISON OF RESULTS*:

At the outer edge (node 14).

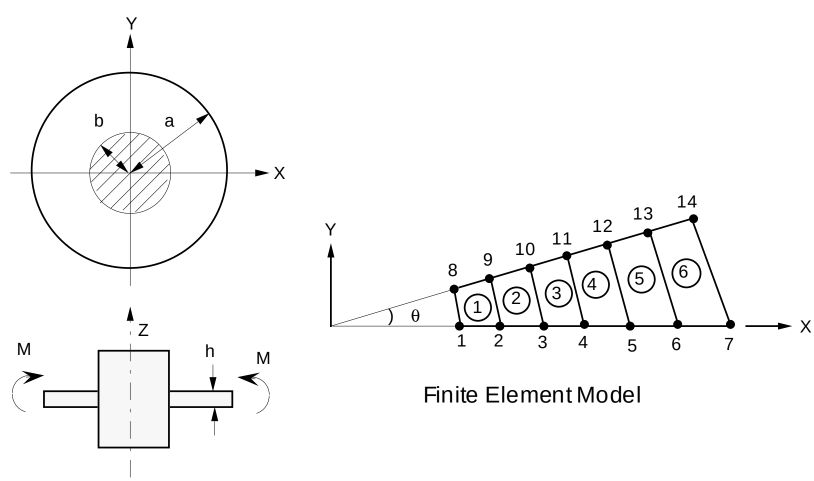
	δ_z, inch	θ_y, rad
Theory	0.0490577	-0.0045089
COSMOS/M	0.0492188	-0.0044562
Difference	0.3%	1.17%

* The above results are tabulated for S44A. Identical results will be obtained for S44B.

	X = 10.86 inch (First Element)		X = 27.2 inch (Sixth Element)	
	Moment in-lb/in	σ_r, psi	Moment in-lb/in	σ_r, psi
Theory	-13.7	1319	-10.1	971.7
COSMOS/M	-13.7	1313	-10.1	972.7
Difference	0%	0.45%	0%	0.10%

* The above results are tabulated for S44A. Identical results will be obtained for S44B.

Figure S44-1



Problem Sketch

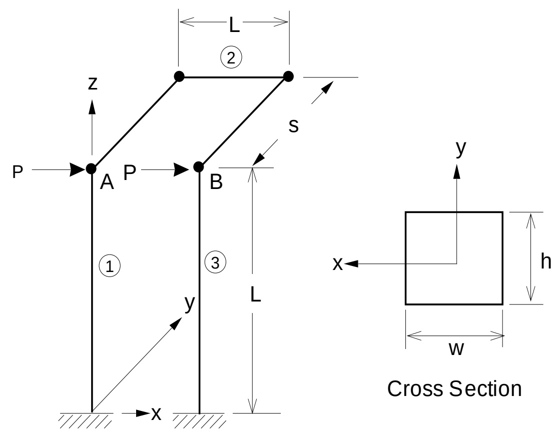
Finite Element Model

S45: Eccentric Frame

TYPE:

Static analysis, beam elements (BEAM3D) and point-to-point constraints (CPCNS command).

Figure S45-1



Problem Sketch and Finite Element Model

REFERENCE:

Laursen H. I., “Structural Analysis,” McGraw-Hill, 1969.

PROBLEM:

Two vertical beams constitute an eccentric portal frame with the aid of 3 horizontal rigid bars. Find the deformations resulting from the horizontal forces.

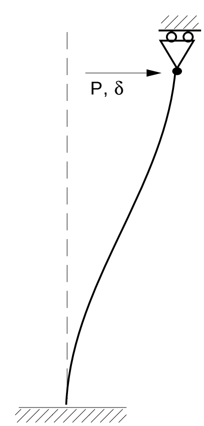
GIVEN:

h	= 1 in	S	= 2.5
W	= 1 in	E	= 1E6 psi
L	= 10 in	P	= 1 lb
I	= 1/12 in^4 (about y and z axis)		

MODELING HINT:

Point-to-point constraint elements are used (i.e., points 2-3, 3-4 and 4-5) to ensure the frame 2-3 - 4-5 is rigid when the horizontal forces are loaded. Each of the beams (1) and (2) will deform as shown:

Figure S45-2



Deflected Shape

COMPARISON OF RESULTS:

Deflection along X-axis at points A and B:

	δ_x (incl
Theory	1.0000E
COSMOS/M	1.0104E
Difference	1%

S46, S46A, S46B: Bending of a Cantilever Beam

TYPE:

Static analysis, plane stress elements (PLANE2D), beam elements (BEAM2D), shell elements (SHELL9) and constraint elements.

PROBLEM:

Calculate the maximum deflection and the maximum rotation of a cantilever beam loaded by a shear force at the free end.

GIVEN:

h	= 1 in	I	= 1/12 in^4	ν	= 0.3
L	= 10 in	E	= lE6 psi	p	= -1 lb

MODELING HINTS: *Continuum-to-Structure Constraint*

Problem 1 (S46):

The plane stress elements are defined by nodes 1 through 12. The beam element is defined by nodes 13 and 14. Each plane stress element is theoretically equivalent to a beam element where I = 1/12 in^4. Node 14 is attached to line 11-12, so displacements and rotations are constrained to be compatible.

Problem 2 (S46A):

Two groups of PLANE2D, plane stress, 8-node elements are coupled together as shown in Figure S46–2 where the geometry and material properties are the same as those in Problem 1. The focus of interest in on the continuum-to-continuum constraint and the location of the primary point which is no longer located at the middle of the 3-point curve, but at any arbitrary position.

Problem 3 (S46B):

Two groups of SHELL9 elements are coupled together as shown in Figure S46–3 where the geometry and material properties are the same as those in Problem 1 and 2. The primary deformation is located in the x-y plane. This problem is provided to verify the accuracy of the structure-to-structure constraint.

ANALYTICAL SOLUTION:

$\delta_y = -pL^3 / 3EI$ $\theta_x = -pL^2 / 2EI$

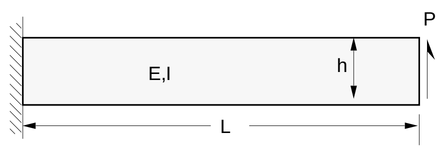
COMPARISON OF RESULTS:

At the free end:

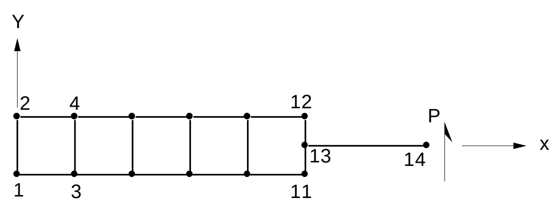
	δ_y inch	θ_z rad
Theory	-4.000E-3	-6.000E-4
Beam Element	-4.000E-3	-6.000E-4
Plane Stress Element	-4.006E-3	-6.000E-4
Beam/Plane Stress Element (S46)	-4.008E-3	-6.000E-4
Plane Stress/Plane Stress Element (S46A)	-4.009E-3	-5.985E-4 *
SHELL9/SHELL9 Element (S46B)	-4.014E-3	-5.990E-4 *

* Computed using displacements at the free end.

Figure S46-1

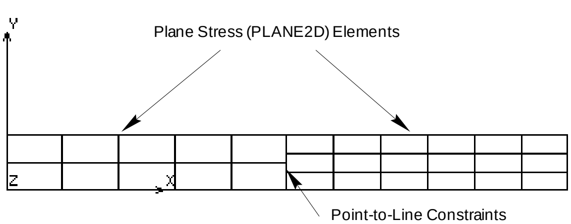


Problem Sketch



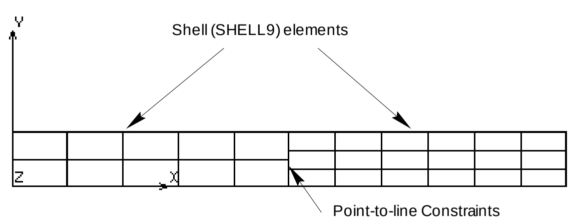
Finite Element Model

Figure S46-2



Finite Element Model - 2

Figure S46-3



Finite Element Model - 3

S47, S47A, S47B: Bending of a Cantilever Beam

TYPE:

Static analysis, SOLID elements, TETRA10 elements, BEAM elements, and point-to-surface constraint elements (attachment).

PROBLEM:

Calculate the maximum deflection and the maximum rotation 0 of a cantilever beam loaded by a shear force at the free end.

GIVEN:

h = 1 in	$I = 1/12\ in^4$	E = 1E6 psi
w = 1 in	(about y and z axes)	$\nu = 0$
L = l0 in		p = 1 lb

MODELING HINTS: *Continuum-to-Structure Constraint*

Problem 1 (S47):

The solid elements are defined by nodes 1 through 24. The beam element is defined by nodes 25 and 26. Each solid element is theoretically equivalent to a beam element where $I = 1/12\ in^4$ about y and z axes. Node 25 is attached to surface 21-22-24-23, so displacements and rotations are constrained to be compatible.

Problem 2 (S47A):

Two groups of SOLID 20-node elements are coupled together as shown in Figure S47-2 where the geometry and material properties are the same as those in Problem 1. The focus of interest is on the continuum-to-continuum constraint and the location of the primary point which is no longer located at the middle of the 8-point surface, but at any arbitrary position.

Problem 3 (S47B):

Two groups of TETRA10 elements are coupled together as shown in Figure S47-3 where the geometry and material properties are the same as those in Problem 1 and 2. This problem is provided to verify the accuracy of the continuum-to-continuum constraint with the primary point located at any arbitrary position of a 6-node surface.

ANALYTICAL SOLUTION:

$\delta = -PL^3 / 3EI$ $\theta = -PL^2 / 2EI$

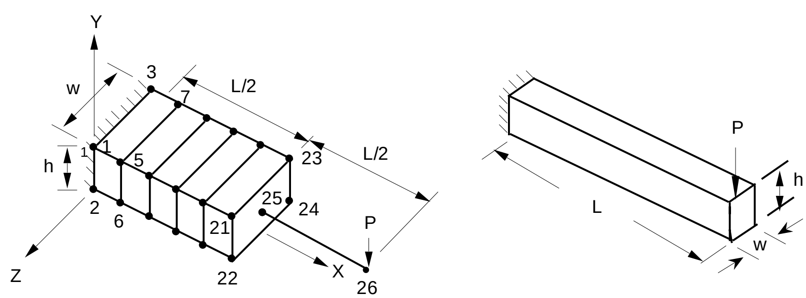
COMPARISON OF RESULTS:

At the free end.

	Deflection δ inch	Rotation θ rad
Theory	-4.000E-3	-6.000E-4
Beam Element	-4.000E-3	-6.000E-4
Plane Stress Element	-4.010E-3	-6.000E-4
Beam/Solid Element (S47)	-4.005E-3	-6.000E-4
Solid/Solid Element (S47A)	-3.986E-3	-5.962E-4 *
Tetra10/Tetra10 Element (S47B)	-3.969E-3	-5.950E-4 *

* Computed using displacements at the free end.

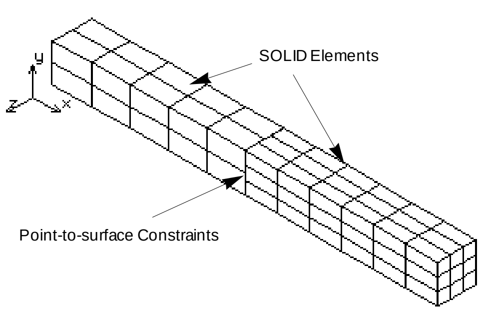
Figure S47-1



Finite Element Model – 1

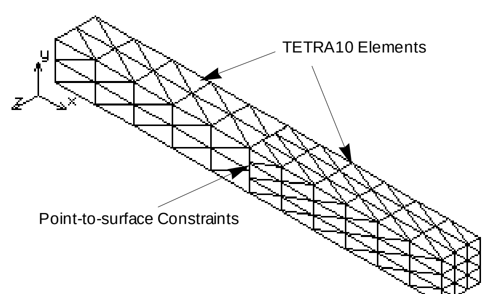
Problem Sketch

Figure S47-2



Finite Element Model - 2

Figure S47-3



Finite Element Model - 3

S48: Rotation of a Tank of Fluid (PLANE2D Fluid)

TYPE:

Static analysis, axisymmetric elements (PLANE2D).

REFERENCE:

Brenkert, Jr., K., “Elementary Theoretical Fluid Mechanics,” John Wiley and Sons, Inc., New York, 1960.

PROBLEM:

A large cylindrical tank is partially filled with an incompressible liquid. The tank rotates at a constant angular velocity about its vertical axis as shown. Determine the elevation of the liquid surface relative to the center (lowest) elevation for various radial positions. Also, determine the pressure p in the fluid near the bottom corner of the tank.

GIVEN:

w = 1 rad/sec

r = 48 in

h = 20 in

ρ = 0.9345E-4 lb-sec^2/in^4

g = 386.4 in/sec^2

b = 30E4 psi

Where:

b = bulk modulus

g = acceleration due to gravity

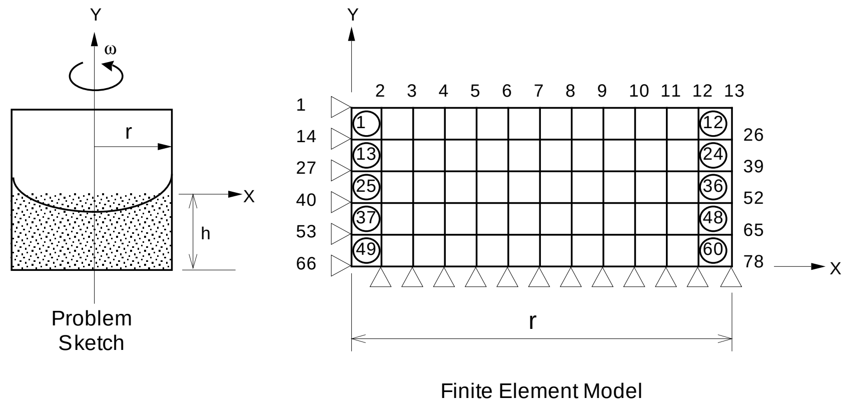
ρ = density

COMPARISON OF RESULTS:

	Displacements * δ_y inch			Pressure (psi)
	Node 4	Node 7	Node 11	Element 60
Theory	-1.86335	0	+1.86335	-0.74248
COSMOS/M *	-1.8627	0	+1.8627	-0.74250
Difference	0.036%	0%	0.03%	0.003%

* After subtracting from the displacement at Node 1 (-1.4798 in)

Figure S48-1


Y
ω
r
X
h
Problem Sketch
Y
2 3 4 5 6 7 8 9 10 11 12 13
1
14
27
40
53
66
1
13
25
37
49
12
24
36
48
60
26
39
52
65
78
X
r
Finite Element Model

S49A, S49B: Acceleration of a Tank of Fluid (PLANE2D Fluid)

TYPE:

Static analysis plane strain (PLANE2D) or SOLID elements.

REFERENCE:

Brenkert, K., Jr., "Elementary Theoretical Fluid Mechanics," John Wiley and Sons, Inc., New York, 1980.

PROBLEM:

Large rectangular tank is partially filled with an incompressible liquid. The tank has a constant acceleration to the right, as shown. Determine the elevation of the liquid surface relative to the zero acceleration elevation for various Y-axis positions. Also, determine the slope of the surface and the pressure p in the fluid near the bottom right corner of the tank.

GIVEN:

a = 45 in/sec^2
b = 48 in
h = 20 in
g = 386.4 in/sec^2
p = 30E4 psi
ρ = 0.9345E-4 lb-sec^2/in^4

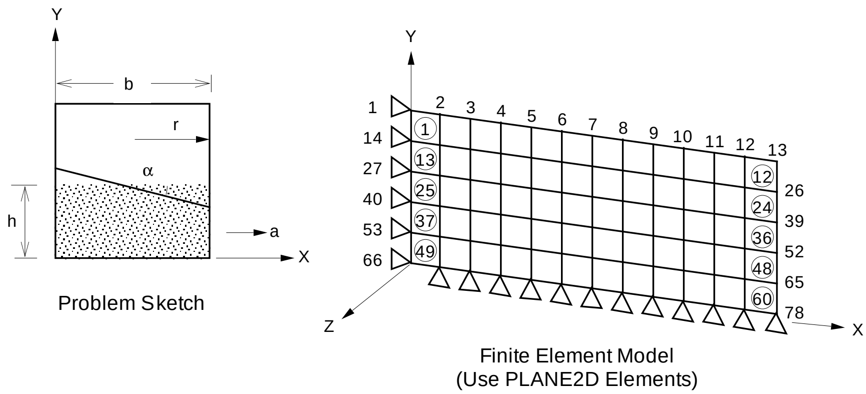
Where:

b = bulk modulus
g = acceleration due to gravity
ρ = density

COMPARISON OF RESULTS:

	Displacements δ_y inch			Pressure (psi)
	Node 3	Node 7	Node 11	Element 60
Theory	-1.86335	0	+1.86335	0.74248
COSMOS/M	-1.8627	0	+1.8627	0.7425
Difference	0.036%	0%	0.03%	0.03%

Figure S49A-1



S50A, S50B, S50C, S50D, S50F, S50G, S50H, S50I: Deflection of a Curved Beam

TYPE:

Static analysis, multi-field elements (4-node PLANE2D, 8-node PLANE2D, SHELL4T, 6-node TRIANG, 8-node SOLID, 20-node SOLID, TETRA4R and SHELL6 elements).

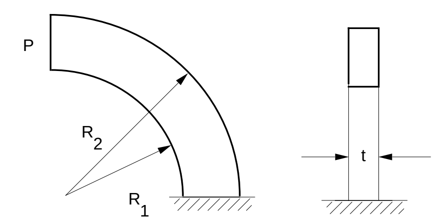
REFERENCE:

Roark, R. J., “Formulas for Stress and Strain,” 4th Edition, McGraw-Hill Book Co., New York, l965, pp. 166.

PROBLEM:

A curved beam is clamped at one end and subjected to a shear force P at the other end. Determine the deflection at the free end.

Figure S50-3



Problem Sketch

GIVEN:

E = 10E6 psi

ν = 0.25

R_1 = 4.12 in

COMPARISON OF RESULTS:

Deflections at free end by theoretical solution is equal to 0.08854 in

Element	COSMOS/M δ_y in^2	Difference (%)
PLANE2D (4-Node) (S50A)	0.08761	1.05%
PLANE2D (8-Node) (S50B)	0.08850	0%
SHELL4T (S50C)	0.08827	0.26%
TRIANG (6-Node) (S50D)	0.07049	11.6%
TETRA4R (4-Node) (S50H)	0.08785	0.8%
SOLID (8-Node) (S50F)	0.08726	1.45%
SOLID (20-Node) (S50G)	0.08848	0.07%
SHELL6 (Curved) (S50I)	0.07498	15.32%
SHELL6 (Assembled) (S50I)	0.062679	29.2%

S51: Gable Frame with Hinged Supports

TYPE:

Static analysis, beam elements (BEAM2D).

REFERENCE:

Valerian Leontovich, “Frames and Arches,” McGraw-Hill Book Co., Inc., New York, 1959, pp. 68.

PROBLEM:

Determine the support reactions for frame shown in the figure.

GIVEN:

L = 16 ft

h = 8 ft

E = 4.32E6 lb/ft^2

f = 6 ft

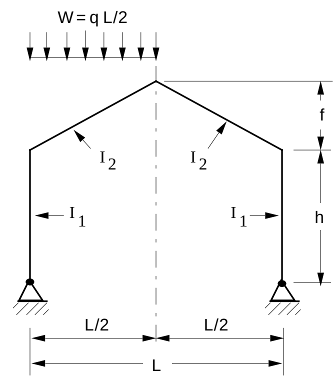
q = 10 lb/ft

$I_{12} = I_{23} = I_{34} = I_{45}$

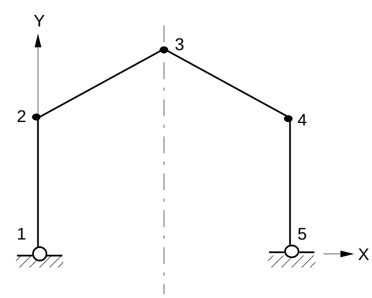
$A_{12} = A_{23} = A_{34} = A_{45}$

Total Load = 4 lbs

Figure S51-1



Problem Sketch



Finite Element Model

MODELING HINTS:

Find load intensity along the frame from

W = (Total load) / q = 4 lb/ft

Then use beam loading commands to solve the problem

COMPARISON OF RESULTS:

Reactions (lb):

Node No.	Theory			COSMOS/M		
	FX	FY	MZ	FX	FY	MZ
1	4.44	30.00	0	4.40	30.00	0
5	-4.44	10.00	0	-4.40	10.00	0

S52: Support Reactions for a Beam with Intermediate Forces and Moments

TYPE:

Static analysis, beam elements (BEAM2D).

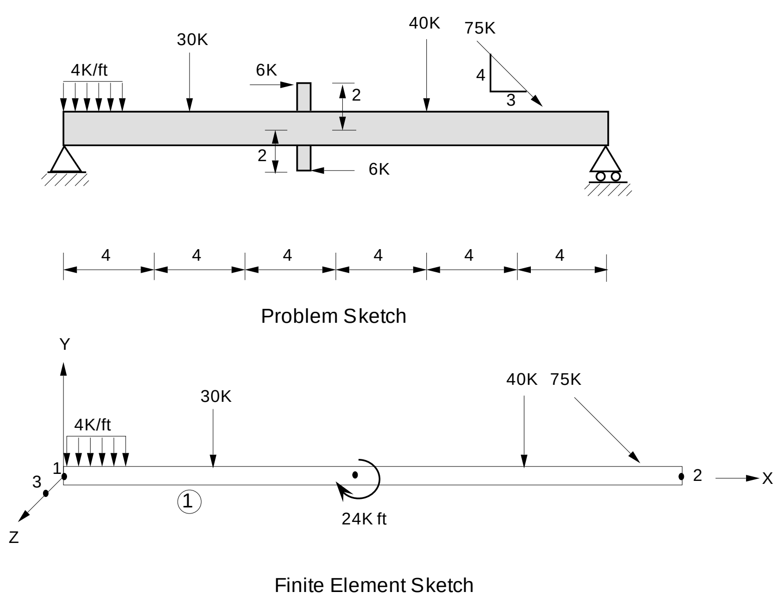
REFERENCE:

Morris, C. H. and Wilbur, J. B., "Elementary Structural Analysis," McGraw-Hill Book Co., Inc., Second Edition, New York, 1960, pp. 93-94.

PROBLEM:

Determine the support reactions for the simply supported beam with intermediate forces and moments.

Figure S52-1



GIVEN:

A $= 0.3472\ ft^2$

$I_y = I_z = 0.02009\ ft^4$

$I_x = 0.04019\ ft^4$

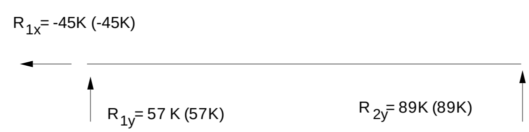
E $= 4320 \times 10^3\ K/ft^2$

NOTE:

The sign convention for the intermediate loads follows the local coordinate system for the beam (defined by the I, J, K nodes).

COMPARISON OF RESULTS:

Figure S52-2



NOTE:

The results obtained with COSMOS/M are compared with those given in reference. The numbers shown in parenthesis are from COSMOS/M.

S53: Beam Analysis with Intermediate Loads

TYPE:

Static analysis, beam elements (BEAM2D).

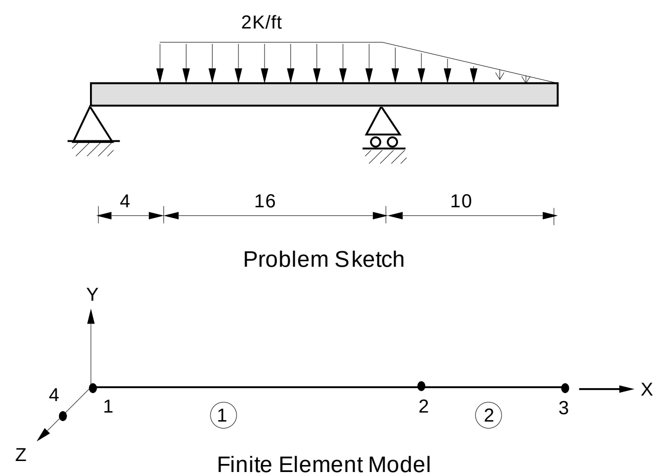
REFERENCE:

Norris, C. H., and Wilbur, J. B., “Elementary Structural Analysis,” 2nd ed., McGraw-Hill Book Co., Inc., 1960, pp. 99.

PROBLEM:

Find the reactions in the support and forces and moments in the beam.

Figure S53-1



GIVEN:

$I_{yy} = I_{zz} = 1 \text{ ft}^4$

$I_{xx} = 2 \text{ ft}^4$

$A = 3.464 \text{ ft}^2$

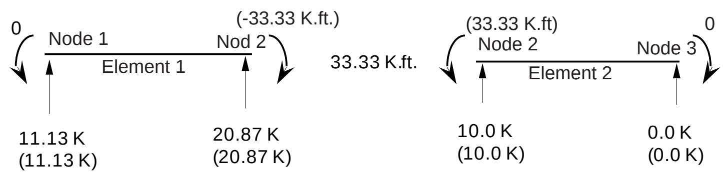
$E = 432 \times 10^4 \text{ k/ft}^2$

NOTE:

The sign convention for intermediate loads, follows the local coordinate system, for the beam (defined by I, J, K nodes).

COMPARISON OF RESULTS:

Figure S53-2



Use the BEAMRESLIST (Results, List, Beam End Force) command to list the results

NOTE:

COSMOS/M results are given in parentheses.

S54: Analysis of a Plane Frame with Beam Loads

TYPE:

Static analysis, beam elements (BEAM2D).

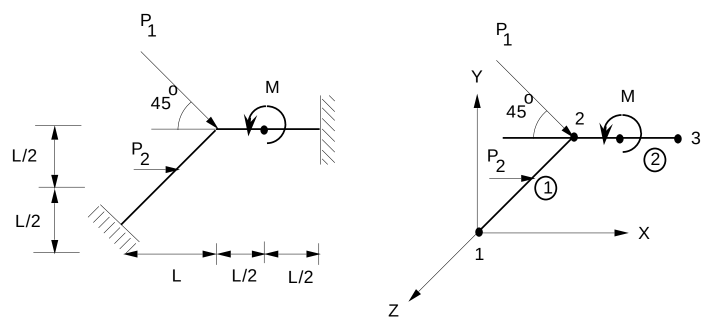
REFERENCE:

Weaver, Jr., W., and Gere, J. M., "Matrix Analysis of Framed Structures," 2nd ed., D. Van Nostrand Company, New York, 1980. pp. 280, 486.

PROBLEM:

Find the deformations and forces in the plane frame subjected to intermediate forces and moments.

Figure S54-1



Problem Sketch

Finite Element Model

GIVEN:

A_x = 0.04 m^2

I_y = I_z = 2 x $10^3 m^4$

I_x = 4 x $10^3 m^4$

E = 200 x 10^6 KN/m^2

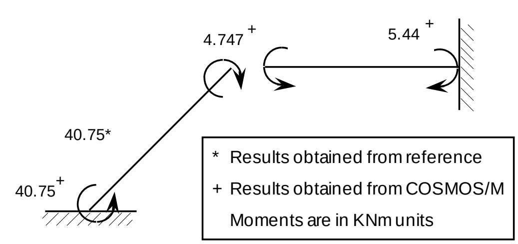
L = 3m

P_1 = 30 x $2^{(1/2)}$ KN

P_2 = 60 KN

M = 180 KN-m

Figure S54-2



S55: Laterally Loaded Tapered Beam

TYPE:

Static analysis, beam element (BEAM3D).

REFERENCE:

Crandall, S. H., and Dahl, N. C., “An Introduction to the Mechanics of Solids,” McGraw-Hill Book Co., Inc., New York, 1959, pp. 342.

PROBLEM:

A cantilever beam of width b and length L has a depth which tapers uniformly form d at the tip to 3d at the wall. It is loaded by a force P at the tip. Find the maximum bending stress at x = L (midspan).

GIVEN:

P = 4000 lb

L = 50 in

d = 3 in

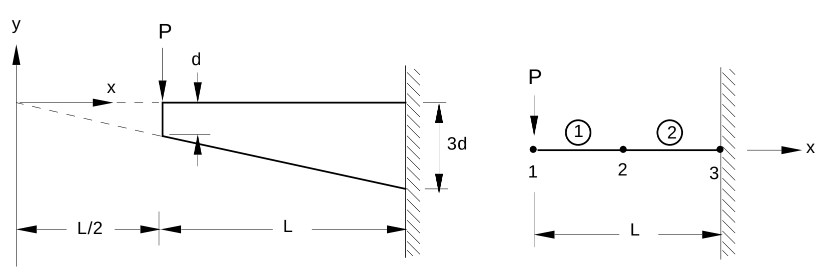
b = 2 in

E = 30E6 psi

COMPARISON OF RESULTS:

	σ_x (psi) (at node 2)
Theory	8333
COSMOS/M	8333

Figure S55-1



Problem Sketch

Finite Element Model

S56: Circular Plate Under a Concentrated Load (SHELL9 Element)

TYPE:

Static analysis, 9-node shell element (SHELL9).

REFERENCE:

Hughes, T. J. R., Taylor, R. L., and Kanoknukulchai, W. A., “Simple and Efficient Finite Element for Plate Bending,” I.J.N.M.E., 11, 1529-1543, 1977.

PROBLEM:

A circular thick plate clamped at the boundary is subjected to a point load at its center. (Shown in Figure S56-1).

Determine the transverse displacement along the radius r.

GIVEN:

E = 1.09E6

ν = 0.3

t = 2 (thickness) in

P = 4 lb

R = 5 in

ANALYTICAL SOLUTION:

$$W_r = \frac{PR^2}{16\pi D}\left[1 - \left(\frac{r}{R}\right)^2 - \frac{2r^2}{R^2}\ \ln\frac{R}{r} - \frac{8D}{KGtR^2}\ \ln\frac{r}{R}\right]$$

Where:

$D = Et^3 / 12(1-\nu^2)$

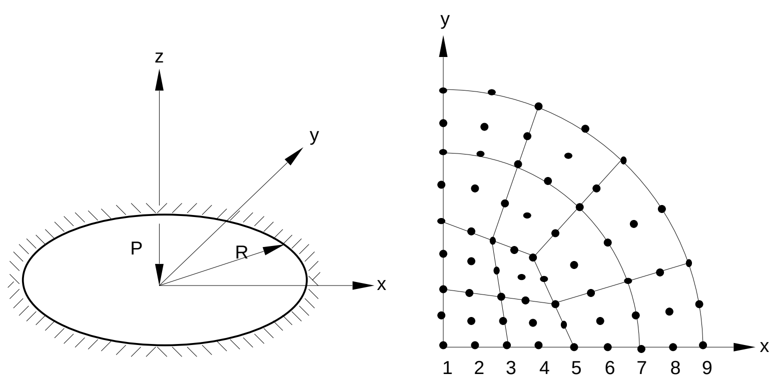
$G = E / 2\,(1+\nu)$

K = 0.8333 (shear correction factor)

COMPARISON OF RESULTS:

Node	r (in)	W_{max} (in) x 10^{-6}	
		Analytical	COSMOS/M
1	0.0	—	-6.469
2	.625	4.185	-4.242
3	1.250	3.1670	-3.166
4	1.875	2.3474	-2.349
5	2.500	1.6366	-1.637
6	3.125	1.0316	-1.033
7	3.750	0.5458	-0.5465
8	4.375	0.1962	-0.1968

Figure S56-1



Problem Sketch

Finite Element Model
(12 Elements)

S57: Test of a Pinched Cylinder with Diaphragm (SHELL9 Element)

TYPE:

Static analysis, 9-node shell element (SHELL9).

REFERENCE:

Dvorkin, E. N., and Bathe, K. J., “A Continue Mechanics Based Four Node Shell Element for General Nonlinear Analysis,” Engineering Computations, 1, 77-78, 1984.

PROBLEM:

A cylindrical shell with both ends covered with rigid diaphragms which allow displacement only in the axial direction of the cylinder is subjected to a concentrated load on the center (shown in the figure below). Determine the radial deflection of point P.

GIVEN:

R = 300 in

L = 600 in

E = 3E6 psi

ν = 0.3

h = 3 (thickness) in

P = 1 lb

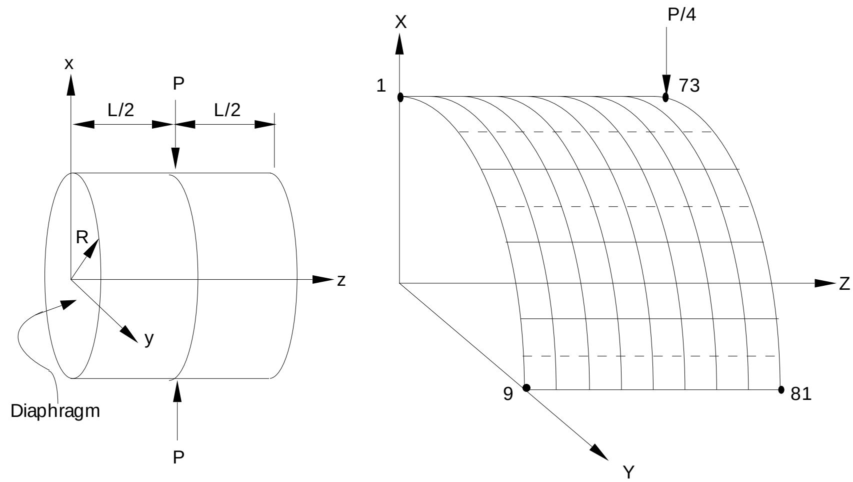
COMPARISON OF RESULTS

	δ_x (inch)
Theory	0.18248E-4
COSMOS/M	0.17651E-4

MODELING HINTS:

Due to symmetry, only one-eighth of the cylinder is modeled. To simulate the rigid diaphragm, on the boundary of the cylinder with z = 0, no rotation along the axial direction (z-axis) is allowed.

Figure S57-1



Problem Sketch

Finite Element Model
(4 x 4)

S58A, S58B, S58C: Deflection of a Twisted Beam with Tip Force

TYPE:

Static analysis, 9-node shell element (SHELL9), 4-node tetrahedral element (TETRA4R).

REFERENCE:

MacNeal, R. H. and Harder, R. L., "A Proposed Standard Set of Problems to Test Finite Element Accuracy," F. E. in Analysis and Design, pp. 3-20, 1986.

PROBLEM:

A twisted beam is subjected to a concentrated load at the tip in the in-plane and out-of-plane directions (shown in the figure below). Determine the deflections coincident with the load.

GIVEN:

L = 12 in

W = 1.1 in

h = 0.32, 0.0032 (thickness) in

F = 1 lb for h = 0.32 and 1e-6 lb for h=0.0032

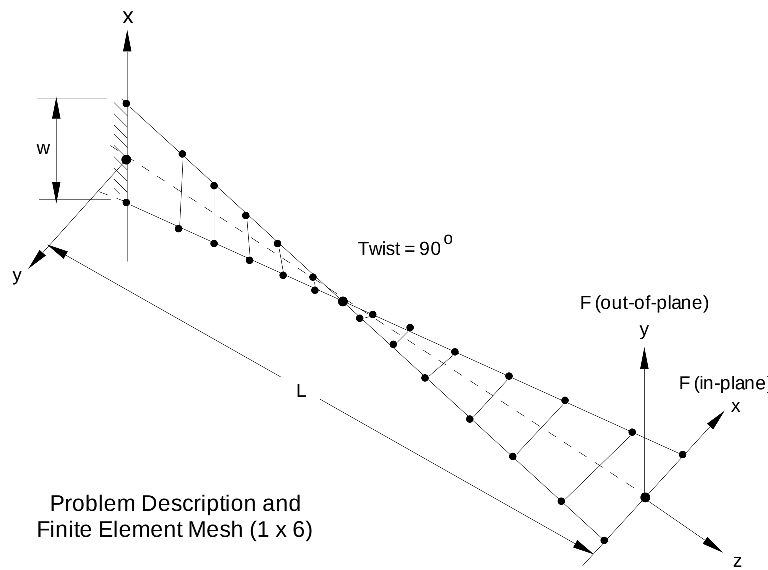
E = 29E6 psi

ν = 0.22

COMPARISON OF RESULTS:

		Deflection in the direction of the force	
Thickness (in)	**Force Direction**	**Theory**	**COSMOS/M**
h = 0.32 in (S58A: SHELL9) Force = 1.0 lb	In-Plane (Load case 1) Out-of-Plane (Load Case 2)	0.5240E-2 0.1754E-2	0.5397E-2 0.1759E-2
h = 0.0032 in (S58B: SHELL9) Force=1e-6 lb	In-Plane (Load case 1) Out-of-Plane (Load Case 2)	0.5256E-2 0.1794E-2	0.4704E-2 0.1255E-2
h = 0.32 (S58C: TETRA4R) Force = 1.0 lb.	In-Plane (Load case 1) Out-of-Plane (Load Case 2)	0.5240E-2 0.1754E-2	0.4967E-2 0.1600E-2

Figure S58-1



Problem Description and
Finite Element Mesh (1 x 6)

S59A, S59B, S59C: Sandwich Square Plate Under Uniform Loading (SHELL9L)

TYPE:

Static analysis, 4- and 9-node composite shell elements (SHELL4L, SHELL9L), solid composite element (SOLIDL).

REFERENCE:

Chang, T. Y., and Sawamiphakdi, K., "Large Deformation Analysis of Laminated Shells by Finite Element Method," Computers and Structures, Vol. 13, pp. 331-340, 1981.

PROBLEM:

A square sandwich plate consisting of two identical facings and an aluminum honeycomb core is subjected to uniform loading as shown in the figure below. Determine the central deflection of the plate at point A.

GIVEN:

Facing:

E = 10.5E6 ksi

ν = 0.3

hf = 0.015 in (thickness)

Core:

E = 0 ksi

a = 25 in

$G_{xz} = G_{yz} = 50$ ksi

P = 9.2311 psi

hc = 1 in (thickness)

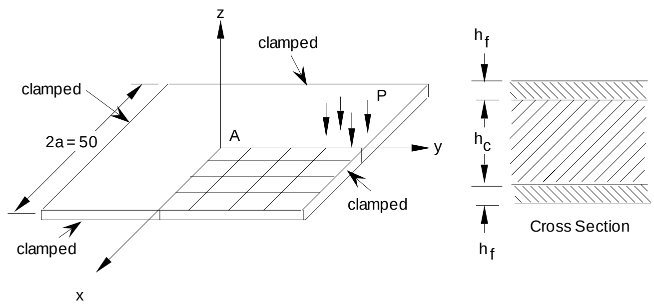
COMPARISON OF RESULTS:

	W_{max} at the Center
Reference	
SHELL4L (S59B)	0.846
SHELL9L (S59A)	0.868
SOLIDL (S59C)	0.849
COSMOS/M	
SHELL4L (S59B)	0.851
SHELL9L (S59A)	0.866
SOLIDL (S59C)	0.849

MODELING HINTS:

Due to symmetry, one quarter of the plate is modeled. To ensure computational stability, a small elastic modulus (E = 1.0E-12) for the core is used.

Figure S59A-1



Problem Sketch and Finite Element Model

S60: Clamped Square Plate Under Uniform Loading

TYPE:

Static analysis, 9-node shell element (SHELL9).

REFERENCE:

Timoshenko, S. P. and Woinowsky-Krieger, S., "Theory of Plates and Shells," 2nd Ed., McGraw Hill, New York, 1959.

PROBLEM:

Determine the maximum deflection (at point A) of a clamped-clamped plate (shown in the figure below) with uniform loading and modeled by a skewed mesh. Various span-to-depth ratios are investigated.

GIVEN:

E = 1E7 psi

ν = 0.3

a = 2 in

q = 1 psi (0.01 psi is used for thickness 0.002)

t = thickness = 0.2, 0.02, and 0.002 in

MODELING HINTS:

Due to symmetry, only one quarter of the plate is modeled.

ANALYTICAL SOLUTION:

$$U_a = 0.00126\, qa^4/D$$

Where:

$$D = Et^3 / 12(1 - \nu^2)$$

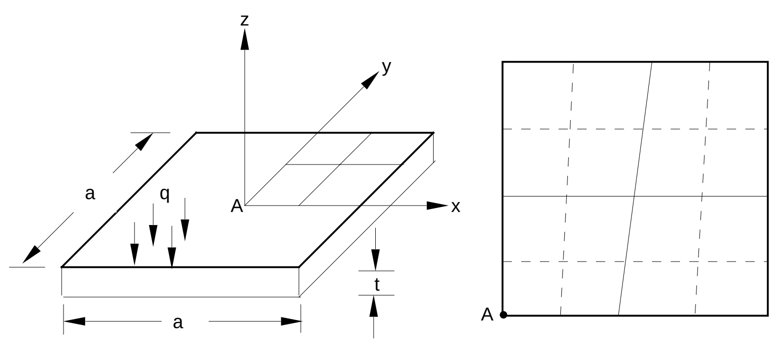
COMPARISON OF RESULTS:

Span/Thickness Ratio *	Deflection (inch)	
	Theory	COSMOS/M
10 (q = 1.0 psi)	-2.7518E-6	-3.4758E-6
100 (q = 1.0 psi)	-2.7518E-3	-3.0649E-3
1,000 (q = 0.01 psi)	-2.7518E-2	-2.79259E-2

* The input file provided (S60.GEO) is for a span/thickness ratio of 10. You need to redefine the thickness for other ratios using the RCONST command.

Better accuracy can be obtained with a finer mesh.

Figure S60-1



Problem Sketch

Finite Element Model
(2 x 2 Skew)

S61: Single-Edge Cracked Bend Specimen, Evaluation of Stress Intensity Factor Using Crack Element

TYPE:

Static analysis, crack element, stress intensity factor, 8-node plane continuum element (PLANE2D).

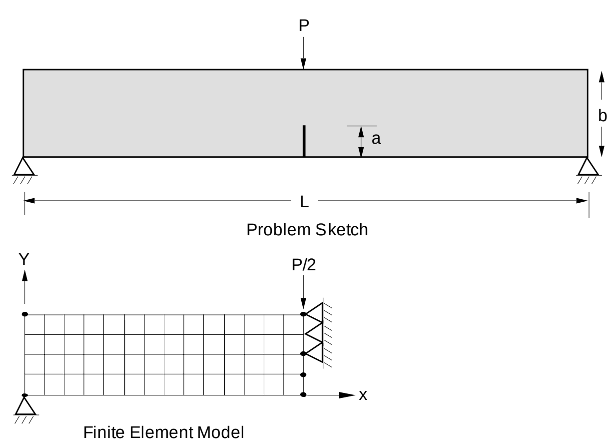
REFERENCE:

Brown, W. F., Jr., and Srawley, J. E., "Plane Strain Crack Toughness Testing of High Strength Metallic Materials," ASTM Special Technical Publication 410, Philadelphia, PA, 1966.

PROBLEM:

Determine the stress intensity factor of a single-edge-cracked bend specimen using the crack element.

Figure S61-1



GIVEN:

E = 30×10^6 psi

ν = 0.3

Thickness = 1 in

a = 2 in

b = 4 in

L = 32 in

P = 1 lb

COMPARISON OF RESULTS:

	K_I
Theory	10.663
COSMOS/M	9.855

S62: Plate with Central Crack

TYPE:

Static analysis, crack stress intensity factor, 8-node plane continuum element (PLANE2D).

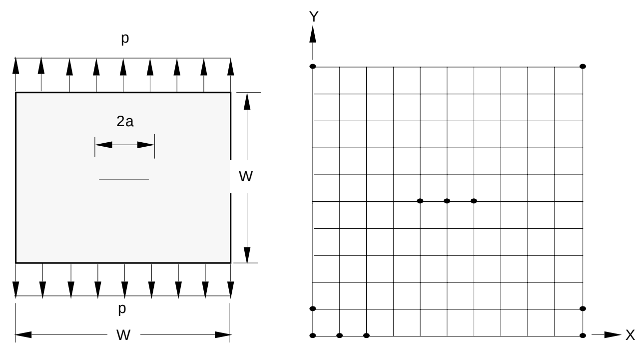
REFERENCE:

Cook, Robert D., and Cartwright, D. J., “Compendium of Stress Intensity Factors,” Her Majesty’s Stationary Office, London, 1976.

PROBLEM:

Determine the stress intensity factor of the center-cracked plate.

Figure S62-1



Problem Sketch

Finite Element Model

GIVEN:

E = $30x\ 10^6$ psi

ν = 0.3

Thickness = 1 in

W = 20 in

a = 2 in

p = 1 lb/in

COMPARISON OF RESULTS:

	K_I
Theory	2.5703
COSMOS/M	2.668

S63: Cyclic Symmetry Analysis of a Hexagonal Frame

TYPE:

Static analysis, cyclic symmetry, truss elements (TRUSS2D).

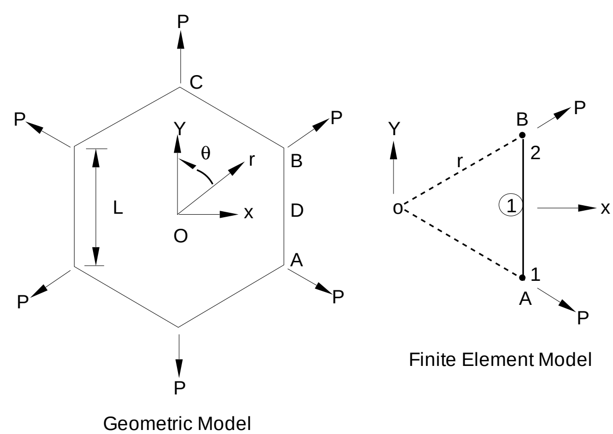
REFERENCE:

Cook, Robert D., "Concepts and Applications of Finite Element Analysis." 2nd Edition, John Wiley & Sons, New York, 1981.

PROBLEM:

The pin-jointed plane hexagon is loaded by equal forces P, each radial from center 0. All lines are uniform and identical. Find the radial displacement of a typical node.

Figure S63-1



GIVEN:

r = 120 in

L = 120 in

A = 10 in^2

P = 3000 lb

E = 30E6 psi

MODELING HINTS:

Taking advantage of the cyclic symmetry of the model and noting that the model displaces radically the same amount at all six nodes, only one element is considered with the radial degree of freedom coupled in the cylindrical coordinate system.

COMPARISON OF RESULTS:

Radial Displacement = 2PL/AE = (3000)(120)/(10)(30E6) = 0.0012 in

	Radial Displacement
Theory	0.0012 in
COSMOS/M	0.0012 in

S64A, S64B: Cyclic Symmetry

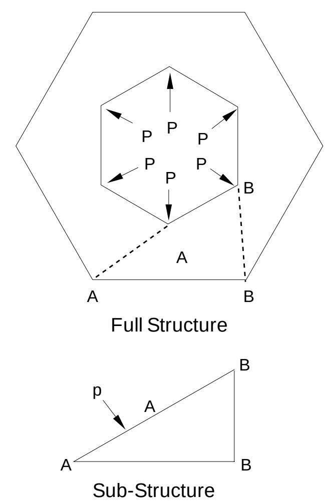
TYPE:

Static analysis, cyclic symmetry, 3-node triangular elements (TRIANG).

REFERENCE:

Cook, Robert D., “Concepts and Applications of Finite Element Analysis.” 2nd Edition, John Wiley & Sons, New York, 1981.

Figure S64-1



PROBLEM:

A hexagonal shaped plate is loaded by a set of radial forces as shown in the figure below. Calculate the deformation of the structure at the point where the load is applied. The plate is considered as a plane stress problem and modeled with 3-node triangular plane elements.

GIVEN:

R = 10 in
t = 1 in
P = 3000 lb
E = 30E6 psi

MODELING HINTS:

This plate is built by combining six sub-structures at 60 degree angles relative to one another. Taking advantage of the cyclic sub-structure may be considered for analysis. Note that for the sub-structure shown, the displacements of nodes along A-A and B-B must be the same in the radial directions. Therefore, these nodes will be coupled radially in the cylindrical coordinate system. All degrees of freedom in the circumferential direction will be fixed.

COMPARISON OF RESULTS:

The problem is solved for both the full structure and the sub-structure with the displacements coming out identical for the corresponding nodes.

Displacement at Point A	X-Displacement	Y-Displacement
Full Model (S64A)	-9.445E-5	-1.636E-4
Cyclic Part (S64B)	-9.450E-5	-1.637E-4

S65: Fluid-Structure Interaction, Rotation of a Tank of Fluid

TYPE:

Static analysis, axisymmetric solid (PLANE2D) and fluid (PLANE2D) elements.

REFERENCE:

S. Timoshenko and S. Woinowsky-Kreiger, “Theory of Plates and Shells,” 2nd Edition, McGraw Hill, New York, 1959, pp. 485-487.

PROBLEM:

A large cylindrical tank is filled with an incompressible liquid. The tank rotates at a constant velocity about its vertical axis as shown. Determine the deflection of the tank wall and the bending and shear stresses at the bottom of the tank wall.

GIVEN:

r = 48 in

h = 20 in

t = 1 in

Fluid:

ρ = 0.9345E-4 lb-sec^2/in^4

b = 30E4 psi

Tank:

E = 3E7 psi

ν = 0.3

ω = 1 rad/sec

g = 386.4 in/sec^2

Where:

b = Bulk modulus

g = Accel. due to gravity

ρ = Density

E = Young's modulus

ν = Poisson’s ratio

COMPARISON OF RESULTS:

		Deflection in x-direction (10^{-6} in)	
Y (in)	**Point**	**Theory**	**COSMOS/M**
20	A	7.381	8.269
16	B	19.544	18.854
12	C	27.805	26.870
8	D	27.161	26.578
4	E	13.604	13.700
0	F	0	0

	Theory	COSMOS/M
σ_{yy}, (max), psi	52.24	40.34
Q_0, lb/in	3.76	3.20

- Note: Compatibility is imposed along the direction normal to the interface using the CPDOF command (LoadsBC, Structural, Coupling, Define DOF Set).

ANALYTICAL SOLUTIONS:

1. Deflection w(y):

$$w = \frac{\gamma r^2}{Et}\left\{(h-y)-e^{-\beta y}\left[h\cos\beta y+\left(h-\frac{1}{\beta}\right)\sin\beta y\right]\right\}\frac{pr^2}{Et}\left[1-e^{-\beta y}\left(\cos\beta y+\sin\beta y\right)\right]$$

$$\beta^4 = \frac{3\left(1-\nu^2\right)}{r^2\,t^2}$$

$\lambda = \rho g$

P = pressure applied on the tank wall due to an angular velocity

2. End Moment M_0:

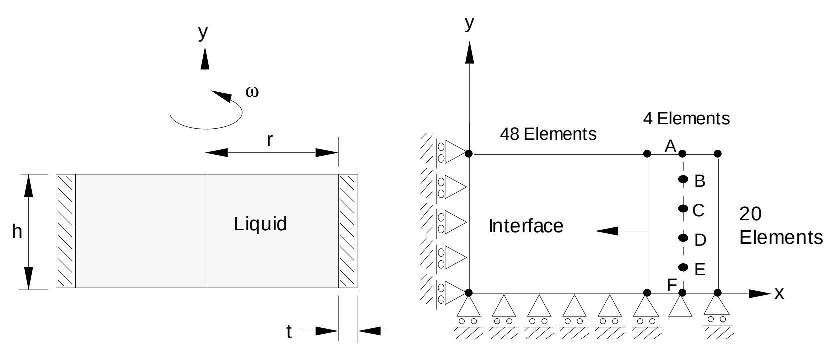
$$M_0 = \left(1-\frac{1}{\beta}\right)\frac{\gamma rht}{\sqrt{12\left(1-\nu^2\right)}} + \frac{prt}{\sqrt{12\left(1-\nu^2\right)}}$$

$$\sigma_{yy}(\max) = \frac{M_0 t}{2I}$$

3. End Shear force Q_0:

$$Q_0 = \frac{\gamma rht}{\sqrt{12\left(1-\nu^2\right)}}\left(2\beta-\frac{1}{h}\right) + \frac{prt}{\sqrt{3\left(1-\nu^2\right)}}\,\beta$$

Figure S65-1



Problem Sketch

Finite Element Model

S66: Fluid-Structure Interaction, Acceleration of a Tank of Fluid

TYPE:

Static analysis, plane strain solid (PLANE2D) and plane fluid (PLANE2D) elements.

REFERENCE:

Timoshenko, S. P., and Gere, James M., "Mechanics of Materials," McGraw Hill, New York, 1971, pp. 167-211.

PROBLEM:

A large rectangular tank is filled with an incompressible liquid. The tank has a constant acceleration to the right, as shown. Determine the deflection of the tank walls and the bending and shear stresses at the bottom of the right tank wall.

GIVEN:

r = 48 in
h = 20 in
t = 1 in

Fluid:

ρ = 0.9345E-4 lb-sec^2/in^4
b = 30E4 psi

Tank:

E = 3E7 psi
ν = 0.3
a = 45 in/sec^2
g = 386.4 in/sec^2

Where:

b = Bulk modulus
g = Gravity Accel.
ρ = Density
E = Young's modulus
ν = Poisson's ratio

COMPARISON OF RESULTS:

Point	Y (in)	W_R (inch)		W_L (inch)	
		Theory	COSMOS/M	Theory	COSMOS/M
A	20	2.137E-3	2.150E-3	6.667E-4	6.761E-4
B	16	1.591E-3	1.602E-3	5.120E-4	5.193E-4
C	12	1.054E-3	1.062E-3	3.552E-4	3.605E-4
D	8	5.564E-4	5.619E-4	1.989E-4	2.024E-4
E	4	1.662E-4	1.689E-5	6.348E-4	6.512E-5
F	0	0	0	0	0

	Theory	COSMOS/M
σ_{yy}, (max), psi (at y = 0)	410.02	386.39
V_0, lb/in (at y = 0)	9.24	8.84*

*This value is calculated by averaging TXY at the nodes located at the bottom of the right wall giving half weight to corner nodes.

- Note: Compatibility is imposed along the direction normal to the interface using the CPDOF command (LoadsBC, Structural, Coupling, Define DOF Set).

ANALYTICAL SOLUTIONS:

1. Deflections of the right wall $W_R(y)$ and the left wall $W_L(y)$:

$$W_R = \frac{P_0 y^2}{120EI}\left(10h^3 - 10h^2 + 5hy^2 - y^3\right) + \frac{P_1 y^2}{24EI}\left(6h^2 - 4hy + y^2\right)$$

$$W_L = \frac{P_0 y^2}{120EI}\left(10h^3 - 10h^2 + 5hy^2 - y^3\right) + \frac{P_2 y^2}{24EI}\left(6h^2 - 4hy + y^2\right)$$

Where:

p_0 = pgh

p_1 = pressure applied on the right wall due to acceleration

p_2 = pressure applied on the left wall due to acceleration

E = $E/(1-v^2)$

2. End Moment M_0

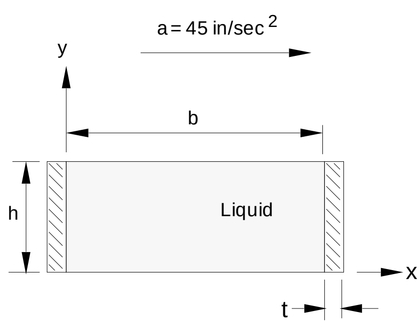
$M_0 = -EI\ d^2W / dy^2$

3. End Shear Force V_0

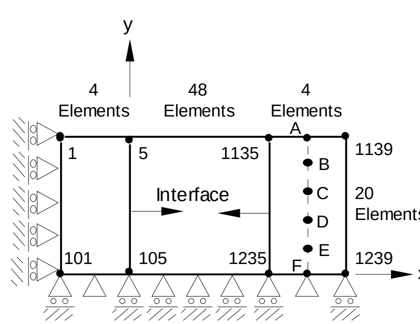
$V_0 = \tau A$

$V_0 = -EI\ d^3W / dy^3$

Figure S66-1



Problem Sketch



Finite Element Model

S67: MacNeal-Harder Test

TYPE:

Static analysis, plane stress quadrilateral p-element (8-node PLANE2D) with the polynomial order of shape functions equal to 5.

PROBLEM:

Calculate the maximum deflection of a cantilever beam loaded by a concentrated end force.

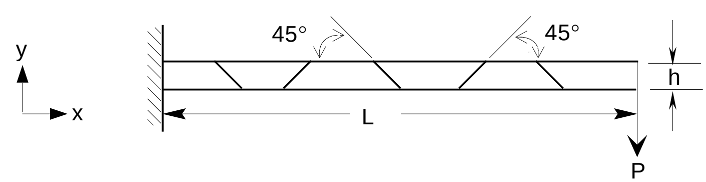
GIVEN:

Geometric Properties:	*Material Properties:*
h = 0.2 in	E = 1×10^7 psi
t = 0.1 in	ν = 0.3
L = 6 in	*Loading:*
I = $2/3 \times 10^{-4}$ in^4	P = 1 lb

COMPARISON OF RESULTS:

	Theory	COSMOS/M
Tip Displacement	0.1081 in	0.10807 in

Figure S67-1



S68: P-Method Solution of a Square Plate with Hole

TYPE:

Static analysis, plane stress quadrilateral (8-node PLANE2D) and triangular (6-node TRIANG) p-elements with the polynomial order of shape functions equal to 5.

PROBLEM:

Calculate the maximum stress of a plate with a circular hole under a uniformly distributed tension load.

GIVEN:

Geometric Properties:

L = 12 in

d = 1 in

t = 1 in

Material Properties:

E = 30 x 10^6 psi

ν = 0.3

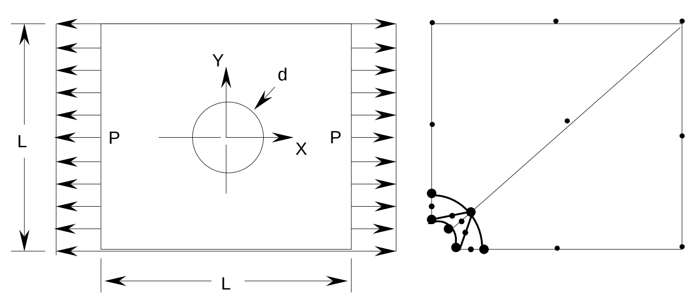
Loading:

p = 1000 psi

COMPARISON OF RESULTS:

	Theory	COSMOS/M (PORD = 5)
Max. Stress in X-Direction	3018	3058 psi

Figure S68-1



S69: P-Method Analysis of an Elliptic Membrane Under Pressure

TYPE:

Static analysis, plane stress triangular p-element (6-node TRIANG).

REFERENCE:

Barlow, J., and Davis, G. A. O., "Selected FE Benchmarks in Structural and Thermal Analysis," NAFEMS Rept. FEBSTA, Rev. 1, October, 1986, Test No. LG1.

PROBLEM:

Calculate the stresses at point D of an elliptic membrane under a uniform outward pressure.

GIVEN:

$E = 210 \times 10^3$ MPa

$\nu = 0.3$

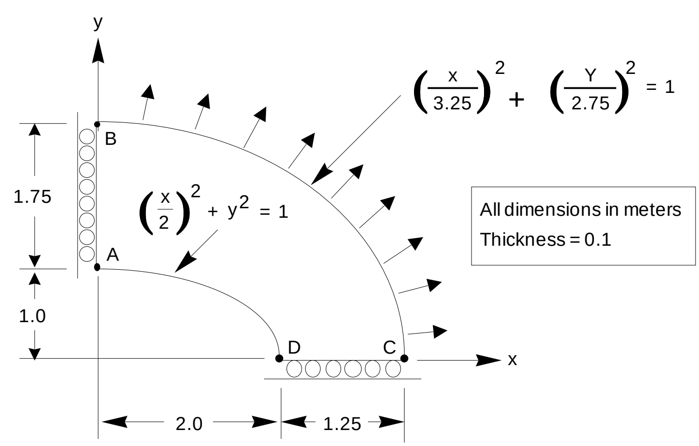
$t = 0.1$

$p = 10$ MPa

COMPARISON OF RESULTS

	σ_y, at Point D
Theory	92.70
COSMOS/M	93.72

Figure S69-1



S70: Thermal Analysis with Temperature Dependent Material

TYPE:

Linear thermal stress analysis, plane continuum element (PLANE2D).

PROBLEM:

A flat plate consists of different material properties through its length. Determine the deflections and thermal stresses in the plate due to uniform changes of temperature equal to 100° F and 200° F.

GIVEN:

t = 0.1 in

x = 0.00001 in/in/° F

ν = 0

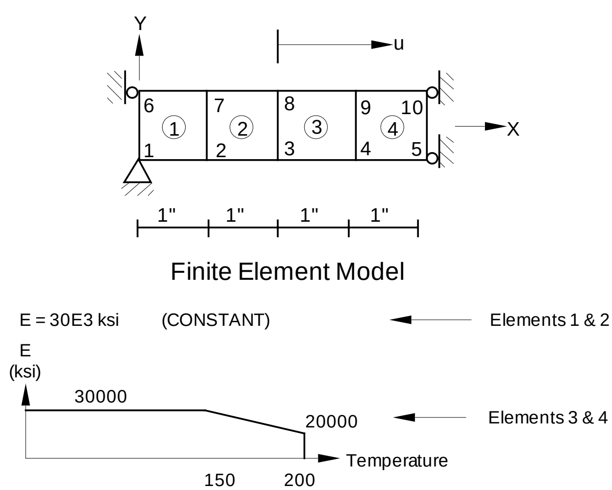
E = 30,000 ksi

COMPARISON OF RESULTS:

	σ_x for All Elements	
	T = 100° F *	**T = 200° F**
Theory	-30 ksi	-48 ksi
COSMOS/M	-30 ksi	-48 ksi

* The temperature in the input file corresponds to T = 200° F. You need to delete the applied temperature using the NTNDEL command and apply temperature of 100° F using the NTND command.

Figure S70-1



S71: Sandwich Beam with Concentrated Load

TYPE:

Static analysis, composite shell element (SHELL4L).

PROBLEM:

Determine the total deflection of the sandwich beam subjected to a concentrated load.

GIVEN:

$E_t = 7000\ N/mm^2$	$E_c = 20\ N/mm^2$	$G_c = 5\ N/mm^2$
$t = 3$ mm	$c = 25$ mm	$d = 28$ mm
$L = 1000$ mm	$b = 100$ mm	$W = 250$ N

THEORY:

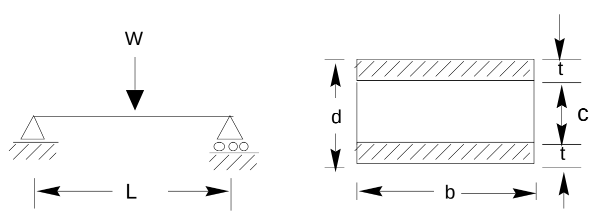
$$D = E_T bt^3/6 + E_T btd^2/2 + E_C bc^3/12 = 8.28 \text{ X } 10^6 \text{ N.mm}^2$$

$$\delta = WL^3/48D + WLc/4bd^2G = 6.2902 + 3.986 = 10.276 \text{ mm}$$

COMPARISON OF RESULTS:

	Midspan Deflection (mm)
Theory	10.276
COSMOS/M	10.323

Figure S71-1



S74: Constant Stress Patch Test (TETRA4R)

TYPE:

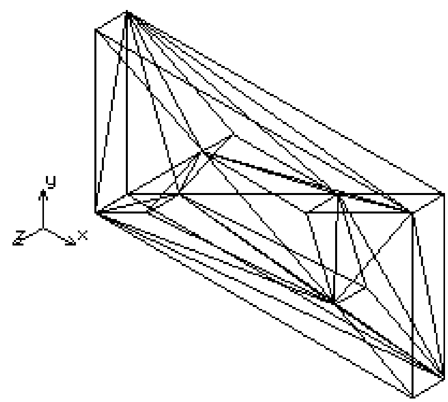
Static analysis, tetrahedral elements (TETRA4, TETRA4R).

PROBLEM:

Constraint displacements at one end and prescribed displacements at the other end of the plate to produce a constant stress state with σ_x = 0.1667E5 and $\sigma_y = \sigma_z = \tau_{xy} = \tau_{yz} = \tau_{zx} = 0$.

Patch test model.

Figure S74-1



Finite Element Model for Patch Test

GIVEN:

Ex = 1E8

υ = 0.25

δx = 0.4E-2

t = 0.024

a = 0.12

b = 0.24

RESULTS:

All the above elements pass the patch test. The nodal stresses show that σ_x = 0.1667E5 and $\sigma_y = \sigma_z = \tau_{xy} = \tau_{yz} = \tau_{zx} = 0$.

S75: Analysis of a Cantilever Beam with Gaps, Subject to Different Loading Conditions

TYPE:

Linear static analysis, beam and gap elements (BEAM2D, GAP).

PROBLEM:

The problem is modeled using BEAM2D elements. Five gap elements with zero gap distances are used. Two different load cases were selected, and the analysis was performed.

GIVEN:

E_{beam} = 30 x 10^6 psi

b = 1.2 in

h = 10 in

L1 = 100 in

L2 = 50 in

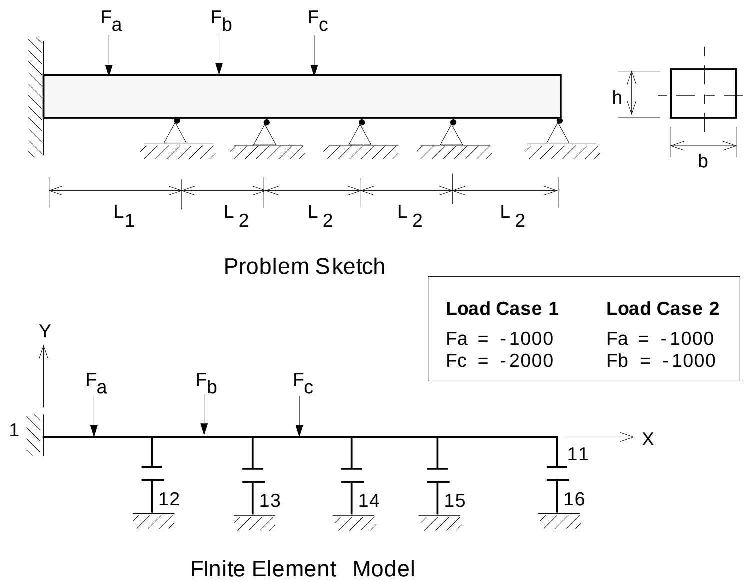
COMPARISONS OF RESULTS:

The deformation state of gaps for each load case agrees with the beam deformed shape corresponding to that load case. The results can be compared with the solution obtained from linear static analysis, where the gaps are removed and the nodes at the closed gaps are fixed.

OBTAINED RESULTS:

		Forces in Gap Elements				
Applied Forces	**Load Case**	**Gap 1**	**Gap 2**	**Gap 3**	**Gap 4**	**Gap 5**
Fa = -1000 Fc = -2000	1	-361.84	-1197.4	-842.11	0	0
Fa = -1000 Fb = -1000	2	-1206.3	-275.0	0	0	0

Figure S75-1


Fa
Fb
Fc
h
b
L1
L2
L2
L2
L2
Problem Sketch
Load Case 1
Fa = -1000
Fc = -2000
Load Case 2
Fa = -1000
Fb = -1000
Y
Fa
Fb
Fc
1
X
11
12
13
14
15
16
FInite Element Model

S76: Simply Supported Beam Subject to Pressure from a Rigid Parabolic Shaped Piston

TYPE:

Linear static analysis, beam, plane and gap elements (BEAM2D, PLANE2D, TRUSS2D and GAP).

PROBLEM:

The shape of the piston is simulated through gap distances. In order to avoid singularities in the structure stiffness, two soft truss elements are used to hold the piston. The problem is analyzed for two different pressure values.

GIVEN:

Gap Distances:

g_1 = g_7 = 0.027 in

g_2 = g_6 = 0.001 in

g_3 = g_5 = 0.008 in

g_4 = 0 in

h = 10 in

b = 1.2 in

k = 1 lb/in

E = 30×10^6 psi

Load case 1: P = 52.5 psi

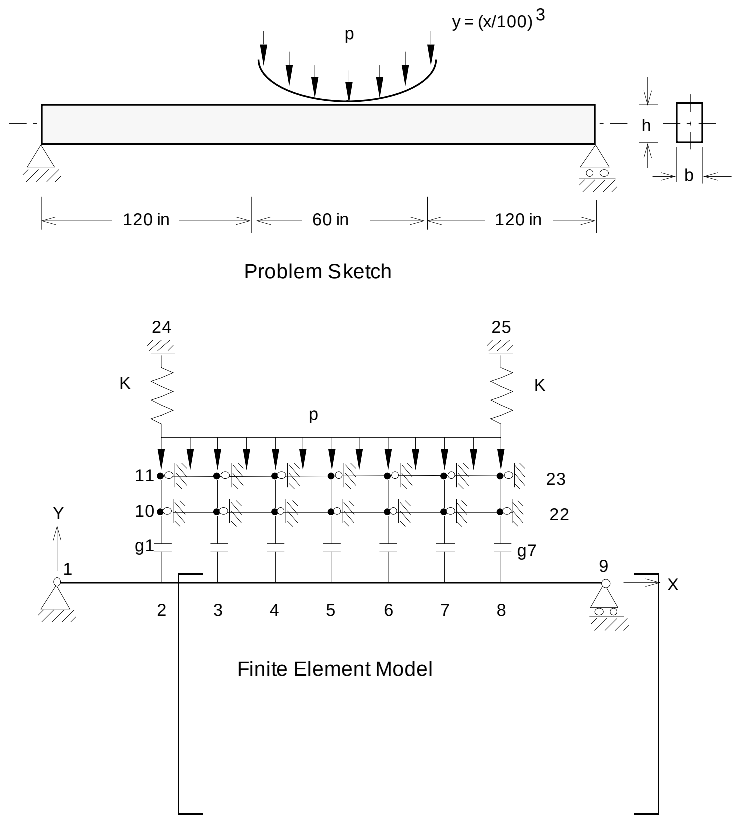
Load case 2: P = 90.8 psi

COMPARISON OF RESULTS:

The forces in the gap elementss at a particular of time are in good agreement with the total force applied to the piston at that time. The deformed shape of the beam for each load case is comaptible with the forces and location of closed gaps for that load case.\

Forces in Gap Elements (lb)				Total Force (Theory)
No. of Closed Gaps	Pressure	Gap Forces	Total	
4	p = 52.5	-215.5 -215.5 -1668.0 -1668.0	-3767	-3780
2	p = 90.8	-3259.0 -3259.0	-6518	-6540

Figure S76-1



S77: Bending of a Solid Beam Using Direct Material Matrix Input

TYPE:

Static analysis, direct material property input, hexahedral solid element (SOLID).

REFERENCE:

Roark, R.J. “Formulas for Stress and Strain”, 4th Edition, McGraw-Hill Book Co., New York, 1965, pp. 104-106.

PROBLEM:

A beam of length L, width b, and height h is built-in at one end and loaded at the free end with a shear force F. Determine the deflection at the free end.

GIVEN:

L = 10 in

E = 30E6 psi

b = 1 in

υ = 0.3

h = 2 in

F = 300 lb

MODELING HINTS:

Instead of specifying the elastic material properties by E and υ, the elastic matrix [D] shown below is provided by direct input of its non-zero terms.

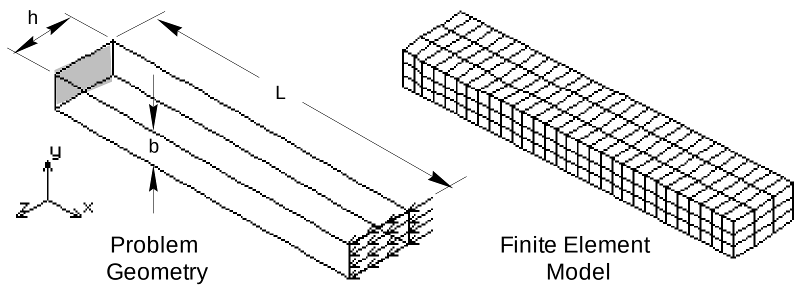
$$[D] \quad \begin{matrix} MC11 & MC12 & MC13 & MC14 & MC15 & MC16 \\ & MC22 & MC23 & MC24 & MC25 & MC26 \\ & & MC33 & MC34 & MC35 & MC36 \\ & & & MC44 & MC45 & MC46 \\ & & & & MC55 & MC56 \\ Sym. & & & & & MC66 \end{matrix}$$

RESULTS:

Displacement in Z-direction at the tip using E and υ, is compared with those obtained with direct input of elastic coefficients in matrix [D].

	Using E and υ	Using Direct Matrix Input
Theory	0.005	0.005
COSMOS/M	0.00496	0.00496

Figure S77-1



S78: P-Adaptive Analysis of a Square Plate with a Circular Hole

TYPE:

P–adaptive analysis, plane stress triangular p-element (TRIANG).

PROBLEM:

Calculate the maximum stress of a plate with a circular hole under a uniform distributed tension load.

GIVEN:

Geometric Properties:

L = 200 in

d = 20 in

E = 30 x 10^6 psi

t = 1 in

Loading:

p = 1 psi

RESULTS:

Nodes: 33, elements: 12, allowable local displacement error: 5%.

Iter. No.	Min p	Max. p	d.o.f.	Energy x 10^{-4}	Max. Displ. x 10^{-6}	Max. Stress	Local Displ. Error %	No. of Sides Not Converged
1	1	1	16	1.692	3.468	1.586	--	22
2	2	2	56	1.701	3.480	2.418	29.544	16
3	2	3	85	1.704	3.489	2.692	12.844	8
4	2	4	100	1.704	3.490	2.817	1.083	0
Ref.	4	4	133	1.706	3.502	2.994	--	--

Figure S78-1

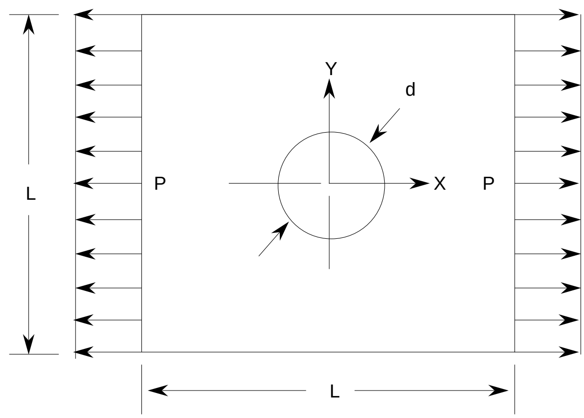
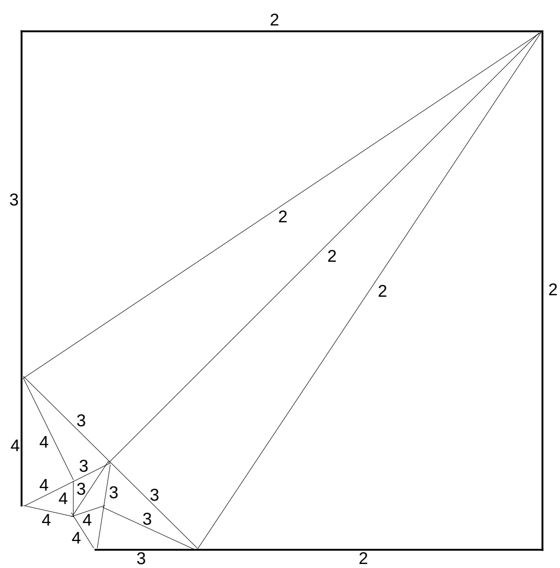


Plate with a Hole

Figure S78-2



Poly nomial O rder for Each Side at Iteration No. 4

S79: Hemispherical Shell Under Unit Moment Around Free Edge

TYPE:

Static linear analysis using the asymmetric loading option (SHELLAX).

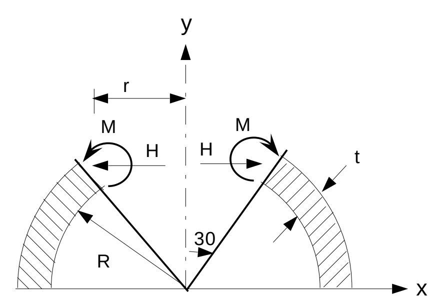
REFERENCE:

Zienkiewicz, O. C., “The Finite Element Method,” 3rd Edition, McGraw Hill Book Co., p 362.

PROBLEM:

Determine the radial displacement of a hemispherical shell under a uniform unit moment around the free edge.

Figure S79-1



Problem Sketch

GIVEN:

R = 100 in
r = 50 in
t = 2 in
E = 1E7 psi
M = 1 in-lb/in
ν (NUXY) = 0.33

MODELING HINTS:

It is important to note that nodal load is to be specified per unit radian which in this case is 50 in-lb/rad.

$$[M_t = \text{Mx Arcx} = \text{Mx Rx } \Psi = 1(50)\ (1) \text{ rad} = 50]$$

Where: Ψ = horizontal angle

COMPARISON OF RESULTS:

	Radial Displacement at Node 31
Theory	1.58E-5 in
COSMOS/M	1.589E-5 in

S80: Axisymmetric Hyperbolic Shell Under a Cosine Harmonic Loading on the Free Edge

TYPE:

Static linear analysis using the asymmetric loading option in SHELLAX.

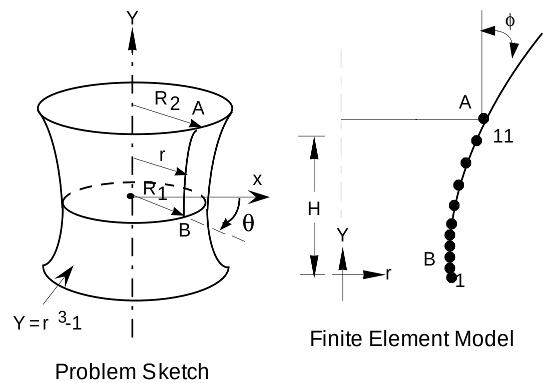
REFERENCE:

NAFEMS, BranchMark Magazine, November, 1988.

PROBLEM:

Determine the stress of an axisymmetric Hyberbolic shell under loading F = cos2 θ on the outward edge, y = 1.

Figure S80-1



GIVEN:

R_1 = 1 m

H = 1 m

tan ϕ = $2^{(-1/2)}$

R_2 = $2^{(1/2)}$ m

E = 210E3 MPa

ν (NUXY)= 0.3

Thickness = 0.01

MODELING HINTS:

Due to symmetry only half of the shell will be modeled. The Cosine load at the free edge will be applied in terms of its x- and y- components, representing the second term of the even function for a Fourier expansion.

COMPARISON OF RESULTS:

The results in the following table correspond to the NXZ component of stress for element 1 as recorded in the output file.

	Shear Stress (y = 0, θ = 45°)
Theory	-81.65 MPa
COSMOS/M	-79.63 MPa

S81: Circular Plate Under Non-Axisymmetric Load

TYPE:

Static linear analysis using the asymmetric loading option in SHELLAX.

REFERENCE:

SHELL4 elements are used for comparison purposes.

PROBLEM:

A circular plate with inner and outer radii of 3 in and 10 in respectively, is subjected to a non-axisymmetric load around outer circumference from $\theta = -54°$ to $\theta = 54°$ perpendicular to the plate surface. The load distribution is:

$$F(\theta) = 5.31\,[1 + \cos(10\,\theta/3)]*10^3$$

GIVEN:

R_i = 3 in

R_0 = 10 in

E = 3E7 psi

t = 1 in

ν (NUXY)= 0.3

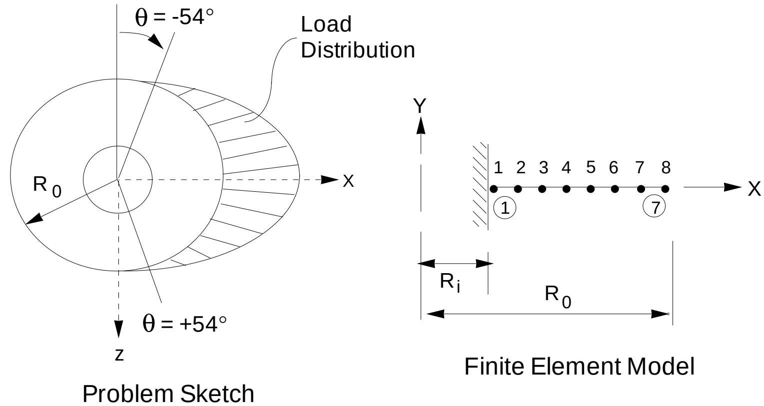
MODELING HINTS:

A total of seven elements are considered in this example. Note that since the load is symmetric about the x-axis, it will be considered only between $\theta = 0°$ and $\theta = 54°$ at 3° intervals, and represented by the even (Cosine) terms of the Fourier expansion. Only the first six (Cosine) terms will be included.

COMPARISON OF RESULTS:

	Displacement of Outer Edges (θ = 180°) in the Axial Direction
SHELLAX	5.80×10^{-4} in
SHELL4	5.62×10^{-4} in

Figure S81-1


θ = -54°
Load
Distribution
R 0
X
θ = +54°
z
Problem Sketch
Y
1 2 3 4 5 6 7 8
1
7
X
R i
R 0
Finite Element Model

S82: Twisting of a Long Solid Shaft

TYPE:

Static analysis, PLANE2D element using asymmetric loading option.

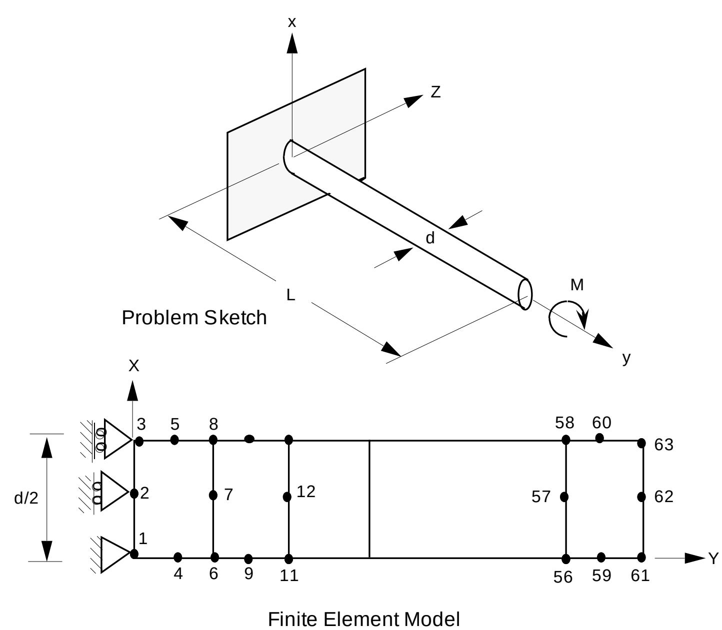
REFERENCE:

Timoshenko, S., “Strength of Materials,” 3rd Edition, D. Van Nostrand Co., Inc., New York, 1956.

PROBLEM:

A long solid circular shaft is built-in at one end and subjected to a twisting moment at the other end. Determine the maximum shear stress, τmax, at the wall due to the moment.

Figure S82-1



GIVEN:

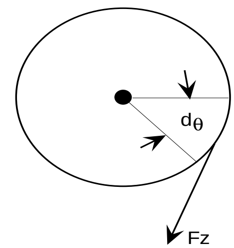
E = 30E6 psi

L = 24 in

d = 1 in

M = -200 in-lb

Figure S82-2



MODELING HINTS:

Since the geometry is axisymmetric about the y-axis, the finite element model, shown in the figure above, is considered for analysis. The effect of the applied moment is calculated in terms of a tangential force integrated around the circumference of the circular rod.

ANALYTICAL SOLUTION:

$$M = -\int_0^{2\pi} F_z \frac{d}{2} \, d\theta = -F_z \pi d \qquad F_z = -\frac{M}{\pi d} = 63.661977 \text{ lb}$$

The load is applied at (node 63) in the z-direction (circumferential). Ux (radial) constraints are not imposed at the wall in order to allow freedom of cross-sectional deformation which corresponds to the assumptions of “negligible shear” stated in the reference.

COMPARISON OF RESULTS:

At clamped edge (node 3).

	Max Shear Stress (psi) τ_{13}
Theory	1018.4
COSMOS/M	1018.4

S83: Bending of a Long Solid Shaft

TYPE:

Static analysis, PLANE2D element using the asymmetric loading option.

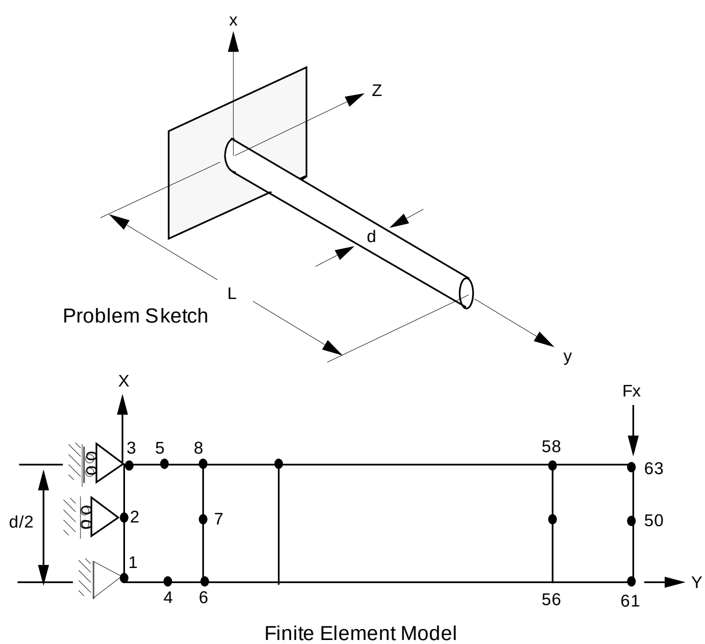
REFERENCE:

Timoshenko, S., “Strength of Materials,” 3rd Edition, D. Van Nostrand Co., Inc., New York, 1956.

PROBLEM:

A long solid circular shaft is built-in at one end and at the other end a vertical force is applied. Determine the maximum axial stress σ_y at the wall and at one inch from the wall due to the force.

Figure S83-1



GIVEN:

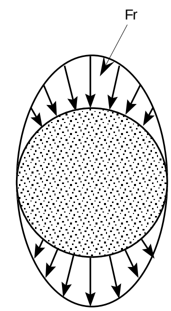
E = 30E6 psi

L = 24 in

d = 1 in

F = -25 lb

Figure S83-2



Load Distribution

MODELING HINTS:

The finite element model is formed as noted in the figure considering the axisymmetric nature of the problem. The force applied at node 63 is calculated based on a Fourier Sine expansion representing its antisymmetric nature.

ANALYTICAL SOLUTION:

$$F = \int_0^{2\pi} F_x \sin^2\theta \, d\theta = F_x \pi \qquad F_x = \frac{F}{\pi} = 7.9577471 \text{ lb}$$

The load is applied at (node 75) in the z-direction (circumferential). Ux (radial) constraints are not imposed at the wall in order to allow freedom of cross-sectional deformation which corresponds to the assumptions of "negligible shear" stated in the reference.

COMPARISON OF RESULTS:

At element 1 and $\theta = 90°$.

	Max Axial Stress (sy psi)	
	y = 0 (Node 3)	y = 1 in (Node 5)
Theory	6111.6	5856.9
COSMOS/M	6115.1	5856.8

S84: Submodeling of a Plate

TYPE:

Static analysis TRIANG element using the submodeling option.

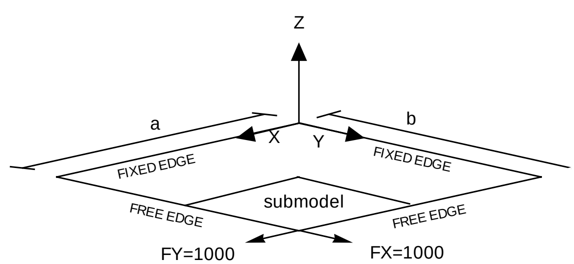
PROBLEM:

Calculate the maximum von Mises stress for a square plate under a concentrated load at one corner. Compare the displacement and stress results from a fine mesh to the results from an originally coarse mesh improved using submodeling.

GIVEN:

a = 25 in
E = 30 E6 psi
b = 25 in
t = 0.1 in
Fx = Fy = 1000 lbs

Figure S84-1



COMPARISON OF RESULTS:

Mesh Type	Max Deflection at Node 1	Max Stress
Coarse Mesh	-0.00131	3810
Coarse Mesh + Submodeling	-0.00156	7626
Fine Mesh and Theory	-0.00156	7620

S85: Plate on Elastic Foundation

TYPE:

Static analysis, SHELL4 plate elements on elastic foundation.

PROBLEM:

A simply supported plate is subjected to uniform pressure P. The full plate is supported by elastic foundation. For small flexural rigidity, the calculated pressure applied to the plate from the foundation approaches the applied external pressures. The flexural rigidity decreases by decreasing the thickness and modulus of elasticity.

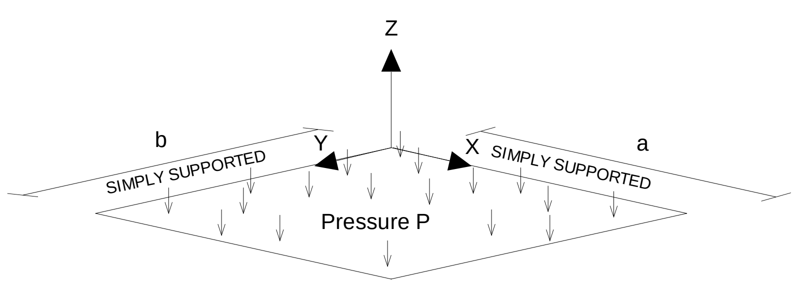
GIVEN:

E = 30 x 10 psi
ν = 0.3
h = 0.01 in
a = 10 in
b = 10 in
P = 10 psi

COMPARISON OF RESULTS:

	Foundation Pressure at Element 200
Theory	-10.0
COSMOS/M	-10.0

Figure S85-1



NOTE:

Foundation pressure is recorded in the output file for each element in the last column of element stress results.

S86: Plate with Coupled Degrees of Freedom

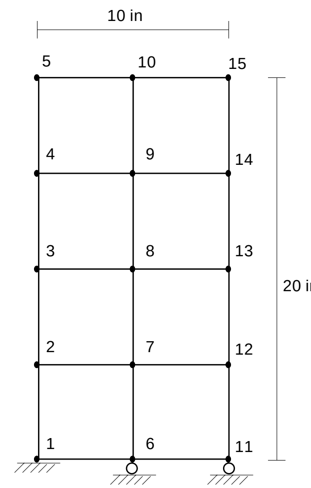
TYPE:

Static analysis, PLANE2D element, coupled degrees of freedom.

PROBLEM:

Determine displacements for the plate shown in the figure below such that translations in the Y-direction are coupled for nodes 5, 10, and 15.

Figure S86-1



GIVEN:

EX = 3.0E10, 3.0E09, and 3.0E8 psi

ν = 0.25

COMPARISON OF RESULTS:

Displacements for the coupled D.O.F.

	Young's Modulus	U5 (Y-Translation at Node 5)	U10 (Y-Translation at Node 10)	U15 (Y-Translation at Node 15)	U10/U5
Theory	3.0E10	5.33333E-7	5.33333E-7	5.33333E-7	1.000
COSMOS/M		5.33333E-7	5.33333E-7	5.33333E-7	1.000
Theory	3.0E09	5.33333E-6	5.33333E-6	5.33333E-6	1.000
COSMOS/M		5.33333E-6	5.33333E-6	5.33333E-6	1.000
Theory	3.0E08	5.33333E-5	5.33333E-5	5.33333E-5	1.000
COSMOS/M		5.33333E-5	5.33333E-5	5.33333E-5	1.000

ANALYTICAL SOLUTION:

$$U10 = U5 = FL/AE$$

S87: Gravity Loading of ELBOW Element

TYPE:

Linear static analysis, ELBOW element with pipe cross-section subjected to gravity loading.

PROBLEM:

Case A: Reduced gravity loading (fixed-end moments ignored)

Case B: Consistent gravity loading (fixed-end moments considered)

GIVEN:

g = -32.2 in/sec^2

EX = 3.0E7 psi

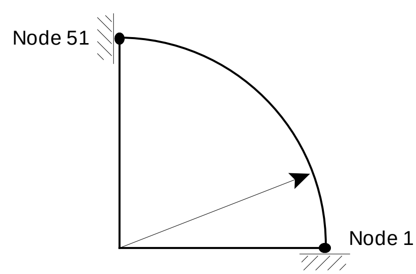
ρ = 7.82

Elbow wall thickness = 0.1 in

Elbow outer diameter = 1.0 in

Elbow radius of curvature = 10.0 in

Figure S87-1



COMPARISON OF RESULTS:

	Y-Translation at Node 51
Case A	-7.42E-3
Case B	-6.41E-3

S88A, S88B: Single-Edge Cracked Bend Specimen, Evaluation of Stress Intensity Factor Using the J-integral

TYPE:

Static analysis, J-integral, stress intensity factor, plane stress conditions.

S88A:	Using 6-node triangular plane element (TRIANG)
S88B:	Using 8-node rectangular plane element (PLANE2D)

REFERENCE:

Brown, W. F., Jr., and Srawley, J. E., “Plane Strain Crack Toughness Testing of High Strength Metallic Materials,” ASTM Special Technical Publication 410, Philadelphia, PA, 1966.

PROBLEM:

Determine the stress intensity factor for a single-edge-cracked bend specimen using the J-integral.

GIVEN:

E = 30×10^6 psi

υ = 0.3

Thickness = 1 in

a = 2 in

b = 4 in

L = 32 in

P = 1 lb

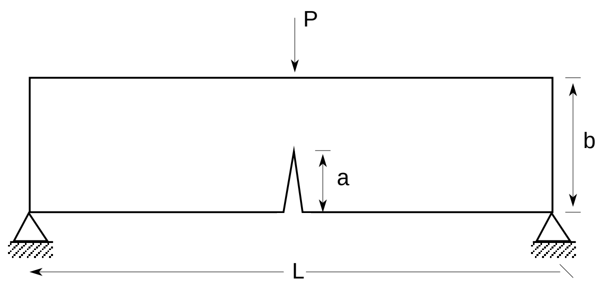
MODELING HINTS:

Three circular J-integral paths centered at the crack tip are considered. Due to symmetry, only one half of the model is modeled.

COMPARISON OF RESULTS

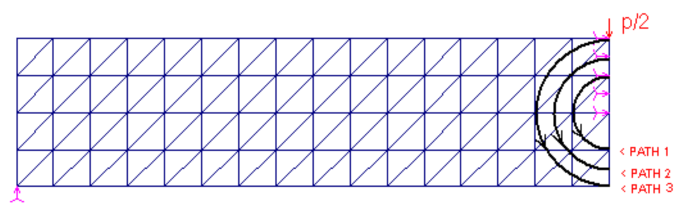
	K_I (TRIANG)	K_I (PLANE2D)
Theory	10.663	10.663
Path 1	9.9544	9.1692
Path 2	10.145	10.974
Path 3	10.240	10.648

Figure S88-1



Full Model

Figure S88-2



S89A, S89B: Slant-Edge Cracked Plate, Evaluation of Stress Intensity Factors Using the J-integral

TYPE:

Static analysis, J-integral, stress intensity factors (combined mode crack), plane strain conditions.

S89A:	Using 6-node triangular plane element (TRIANG)
S89B:	Using 3-node triangular plane element (TRIANG)
S89C:	Using 8-node rectangular plane element (PLANE2D)
S89D:	Using 4-node rectangular plane element (PLANE2D)

REFERENCE:

Bowie, O. L., “Solutions of Plane Crack Problems by Mapping Techniques,” in Mechanics of Fracture I, Methods of Analysis and Solutions of Crack Problems (Ed G.C. Shi), pp. 1-55, Noordhoff, Leyden, Netherlands, 1973.

PROBLEM:

Determine the stress intensity factor for both modes of fracture (opening and shearing) for a rectangular plate with an inclined edge crack subjected to uniform uniaxial tensile pressure at the two ends.

GIVEN:

σ = 1 psi

h = 2.5 in

W = 2.5 in

a = 1 in

E = 30 x 10^6 psi

υ = 0.3

Thickness =1 in

ϕ = 45°

MODELING HINTS:

The full part has to be modeled since the model is not symmetric with respect to the crack. There is no restriction in the type of the mesh to be used and the mesh could be either symmetric or non-symmetric with respect to the crack. However, the nodes in the two sides of crack should not be merged in order to model the rupture area properly.

COMPARISON OF RESULTS:

		K_I	K_{II}
	Reference	**1.85**	**0.880**
6-node element (S89A.GEO)	Path 1 Path 2	1.82 1.82	0.876 0.877
3-node element (S89B.GEO)	Path 1 Path 2	1.76 1.77	0.835 0.873
8-node element (S89C.GEO)	Path 1 Path 2	1.80 1.79	0.872 0.874
4-node element (S89D.GEO)	Path 1 Path 2	1.73 1.71	0.879 0.845

Figure S89-1

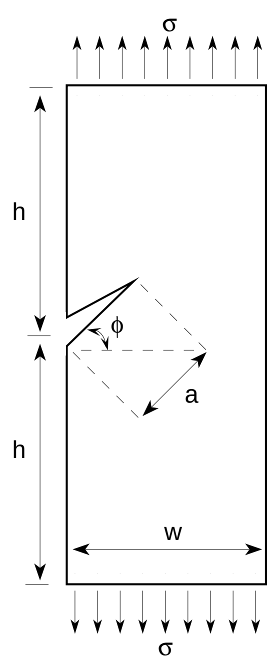
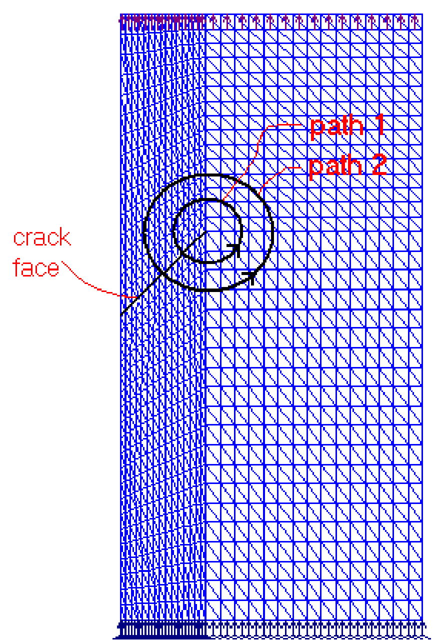


Figure S89-2



S90A, S90B: Penny-Shaped Crack in Round Bar, Evaluation of Stress Intensity Factor Using the J-integral

TYPE:

Static analysis, J-integral, stress intensity factor, axisymmetric geometry.

S90A: Using 8-node rectangular plane element (PLANE2D)

S90B: Using 6-node triangular plane element (TRIANG)

REFERENCE:

Tada, H. and Irwin, R., "the stress analysis of cracks Handbook," Paris Productions, Inc., pp. 27.1, St. Louis, MI, 1985.

PROBLEM:

Determine the stress intensity factor for a circular crack inside a round bar subjected to uniform axial tensile pressure at the two ends.

GIVEN:

$\sigma = 1$ psi

H = 25 in

R = 5 in

a = 2.5 in

E = 30×10^6 psi

$\gamma = 0.28$

MODELING HINTS:

Since the model is symmetric with respect to the crack, therefore only one-half of the model (lower half here) is needed for the analysis.

COMPARISON OF RESULTS:

		K_I
	Reference	**1.94**
8-node element (S90A.GEO)	Path 1 Path 2	1.90 1.91
6-node element (S90B.GEO)	Path 1 Path 2	1.89 1.90

Figure S90-1

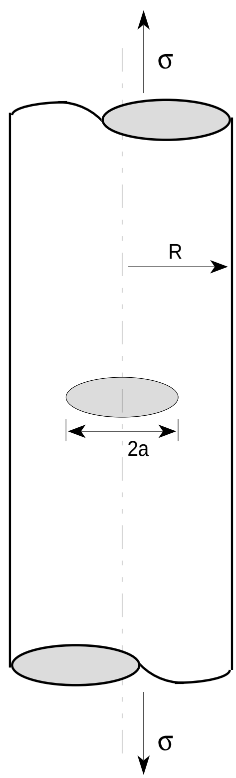
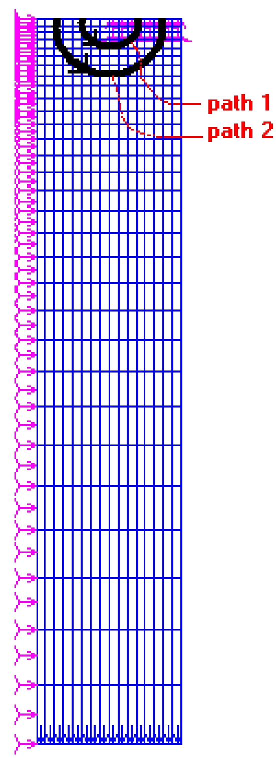


Figure S90-2



S91: Crack Under Thermal Stresses, Evaluation of Stress Intensity Using the J-integral

TYPE:

Static analysis, thermal loading, J-integral, stress intensity factor, plane strain conditions.

REFERENCE:

Wilson, W. K. and Yu, I. W., "The Use of the J-integral in Thermal Stress Crack Problems," International Journal of Fracture, Vol. 15, No. 4, August 1979.

PROBLEM:

Determine the stress intensity factor for an edge crack strip subjected to thermal loading. The strip is subjected to a linearly varying temperature through its thickness with zero temperature at midthickness and temperature To at the right edge (x=w/2). The ends are constrained.

GIVEN:

L = 20 in

w = 10 in

a = 5 in

E = 30×10^6 psi

γ = 0.28

α = 7.4×10^{-6} in/in-° F

T_o = 10 ° F

MODELING HINTS:

Due to symmetry, only one-half of the geometry is modeled (lower half in this problem).

COMPARISON OF RESULTS:

$$\beta = K_I / \left(\sigma_T \sqrt{\pi a}\right), \quad \sigma_T = E\alpha T_o / (1 - \gamma)$$

where: $\beta = K_1/12220.27$

	K_I	K_I/β
Reference		0.5036*
Path 1 **Path 2**	6141.4 6176.3	0.5034 0.5054

*Average value of the five paths in the reference

Figure S91-1

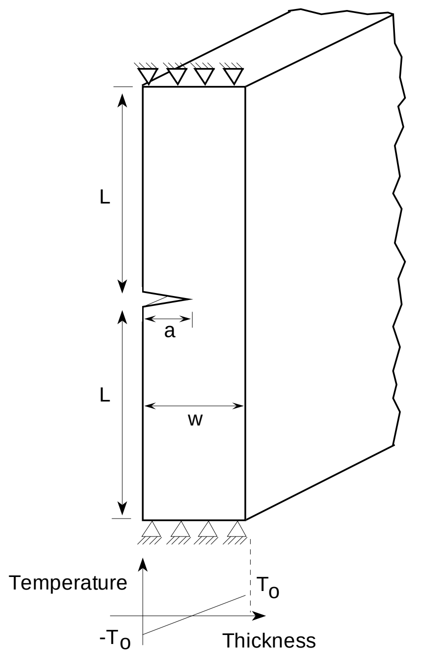
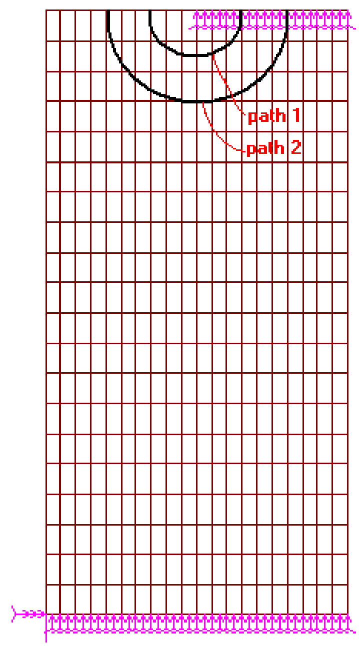


Figure S91-2



S92A, S92B: Simply Supported Rectangular Plate, Using Direct Material Matrix Input

TYPE:

Static analysis, direct material input, SHELL3L element.

REFERENCE:

Timoshenko, S. P. and Woinowsky-Krieger, “Theory of Plates and Shells,” McGraw-Hill Book Co., 2nd edition, pp. 143-120, 1962.

PROBLEM:

Calculate the deflection and stresses at the center of a simply supported plate subjected to a concentrated load F.

GIVEN:

E	$= 30 \times 10^6$ psi	h	= 1 in
G_{xy}	$= G_{yz} = G_{xz} = 11.538 \times 10^6$ psi	a	= b = 40 in
ν	= 0.3	F	= 400 lbs

MODELING HINTS:

Instead of specifying the elastic properties by E and ν, the elastic matrix [D] shown below (in the default element coordinate system) is provided by direct input of its non-zero terms.

$$[D] = \begin{bmatrix} MC11 & MC12 & 0 & 0 & 0 \\ & MC22 & 0 & 0 & 0 \\ & & MC44 & 0 & 0 \\ & & & MC55 & 0 \\ & Sym & & & MC66 \end{bmatrix}$$

where, [D] relates the element strains to the element stresses according to Hook's law:

$$\left\{\sigma_x, \sigma_y, \sigma_{xy}, \sigma_{yz}, \sigma_{xz}\right\}^T = [D]\left\{\varepsilon_x, \varepsilon_y, \gamma_{xy}, \gamma_{yz}, \gamma_{xz}\right\}$$

Note that the [D] matrix is reduced to a 5x5 matrix from the general form of 6x6 matrix, by considering the fact that $\sigma_z = 0$ for shell element, thus eliminating the third row and column of the general [D] matrix.

Considering an isotropic property, the terms of [D] matrix are:

$$MC11 = MC22 = \frac{E}{1-\nu^2} \cong 32{,}967{,}000.$$

$$MC12 = \frac{E\nu}{\left(1-\nu^2\right)} \cong 9{,}890{,}000.$$

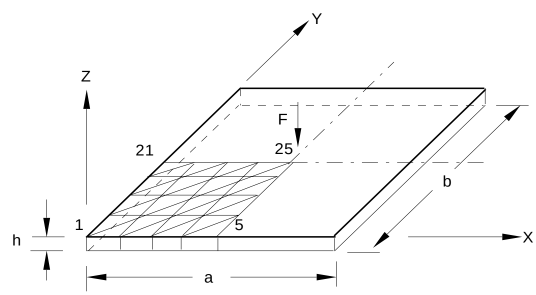
$$MC44 = G_{xy} = \frac{E}{2\left(1+\nu\right)} \cong 11{,}538{,}000.$$

$$MC55 = K_1\, G_{yz} \cong 1{,}159{,}600$$

$$MC66 = K_2\, G_{xz} \cong 1{,}159{,}600$$

The terms K_1 and K_2 are shear correction factors which are chosen to match the plate theory with certain classical solutions and are functions of thickness and material properties. When you input regular material properties (E, ν), the shear factors are evaluated internally in the program as $K_1 = K_2 = 0.1005$ (as in S92B). For the sake of consistency, the same values are used for the evaluation of MC55 and MC66 in S92A.

Figure S92-1



Problem Sketch and Finite Element Model

Due to symmetry in geometry and load, only a quarter of the plate is modeled.

COMPARISON OF RESULTS:

Maximum displacement (in Z-direction) at the tip of the plate (Node 25) using E and ν (S92B) is compared with the result obtained from direct input of the elastic coefficients in matrix [D] (S92A).

	Theory	Using Direct Matrix Input (S92A)	Using E and ν (S92B)
Maximum UZ (in)	-0.0270	-0.02746	-0.02746

S93: Accelerating Rocket

TYPE:

Static analysis, inertia relief, PLANE2D element, (axisymmetric option).

PROBLEM:

A cylinder is accelerating under unbalanced external loads. Find the induced *counter-balance acceleration* and the amount by which the cylinder will be shortened.

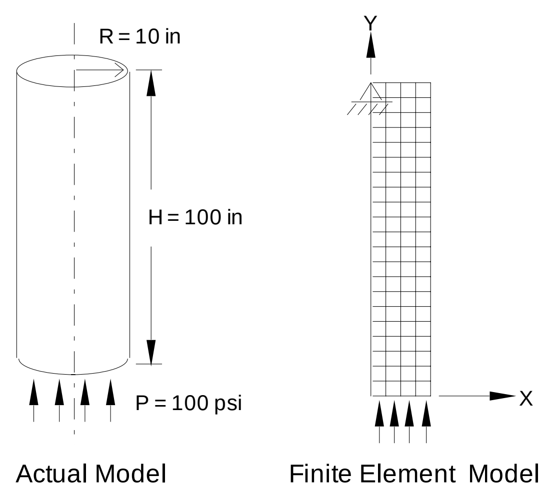
GIVEN:

EX = 3.E7 psi

γ = 0.28 lb sec^2/in^4

ρ = 7.3E-4

Figure S93-1

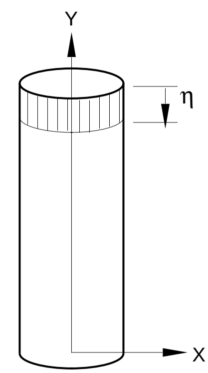


MODELING HINTS:

To avoid instability in FEA solution, one node should be constrained in Y-direction. A node on the top end of the cylinder is selected for that purpose rather than on the bottom end. Constraining any node on the surface where the pressure is applied eliminates the components of the load of that node and hence causes inaccuracy in the solution.

ANALYTICAL SOLUTION:

Figure S93-2



a) The induced counter-balance acceleration:

$$F + Ma = 0$$

$$P \pi R^2 = -\rho \pi R^2 Ha$$

$$a = -\frac{P}{\rho H} = \frac{-100}{0.00073\,(100)} = -1370$$

b) Length shortening

$$\delta = \int_o^H \varepsilon d\eta$$

$$\varepsilon = \frac{\sigma}{E} = \frac{\rho a \eta}{E}$$

$$\delta = \frac{\rho a}{E}\int_o^H \eta d\eta = \frac{\rho a H^2}{2E} = 0.0001667$$

COMPARISON OF RESULTS:

	Acceleration (a)	Displacement u_y at Node 5
Theory	-1370	0.0001667
COSMOS/M	-1370*	0.0001669

* See output file.

S94A, S94B, S94C: P-Method Solution of a Square Plate with a Small Hole

TYPE:

Static analysis using the p-method. S94A: plane stress triangular elements (TRIANG). S94B: plane stress quadrilateral elements (PLANE2D). S94C: Tetrahedral elements (TETRA10).

PROBLEM:

Calculate the maximum stress of a plate with a circular hole under a uniformly distributed tension load. Use strain energy to adapt the p-order.

GIVEN:

Geometric Properties:

L = side of the plate = 10.00 in

d = diameter of the hole = 1.00 in

t = thickness of the plate = 0.25 in

Material Properties:

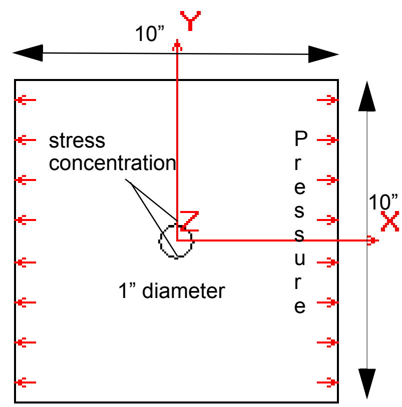
E = 3.0E7 psi

ν = 0.3

Loading:

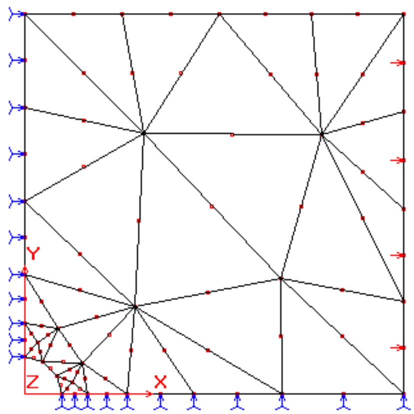
P = 100 psi

Figure S94-1: The Plate with a Hole Model

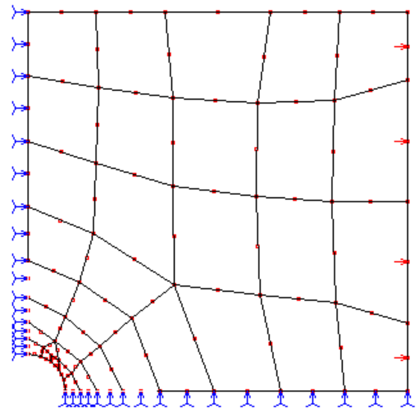


- A coarse mesh is intentionally used to demonstrate the power of the p-method

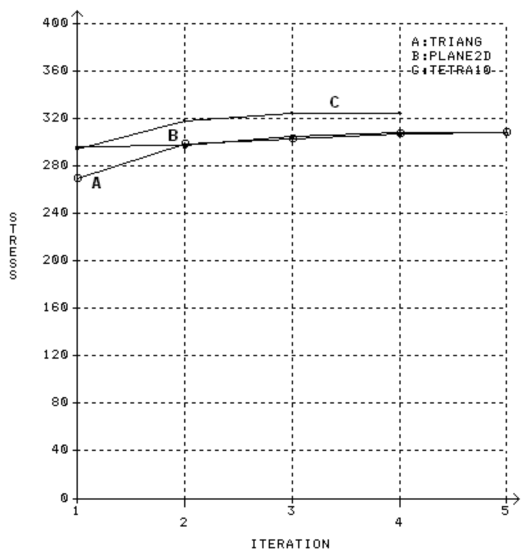
Figure S94-2 Meshed Quarter of the Plate.



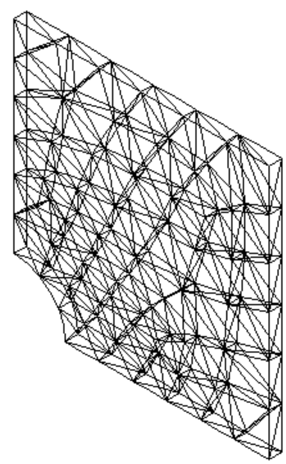
S94A: TRIANG Elements



S94B: PLANE2D elements



Convergence Plots Using Different Element Types



S94C: TETRA10 Elements

COMPARISON OF RESULTS:

	Theory	COSMOS/M	Relative Error
Max. Stress in X-Direction (TRIANG)	300 psi	308 psi (p-order = 4)	2.7%
Max. Stress in X-Direction (PLANE2D)	300 psi	308 psi (p-order = 3)	2.7%
Max. Stress in X-Direction (TETRA10)	300 psi	323 psi (p-order = 5)	7.7%

Reference:

Walter D. Pilkey, "Formulas For Stress, Strain, and Structural Matrices," Wiley-Interscience Publication, John Wiley & Sons, Inc., 1994, pp. 271.

S95A, S95B, S95C: P-Method Solution of a U-Shaped Circumferential Groove in a Circular Shaft

TYPE:

Static analysis, axisymmetric triangular (6-node TRIANG) and quadrilateral (8-node PLANE2D) p-elements with the polynomial order of shape function equal to 8.

PROBLEM:

Calculate the maximum stress of a circular shaft with a U-shape circumferential groove under a uniformly distributed tension load. P-order is adapted by checking strain energy of the system.

GIVEN:

Geometric Properties:

L = 0.9 in

D = 2 in

d = 0.2 in

Material Properties:

E = 3.0E7 psi

ν = 0.3

Loading:

P = 100 psi

Figure S95-1: The Circular Shaft Model

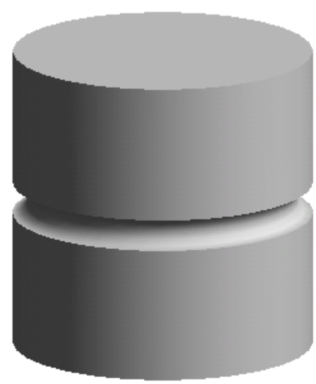
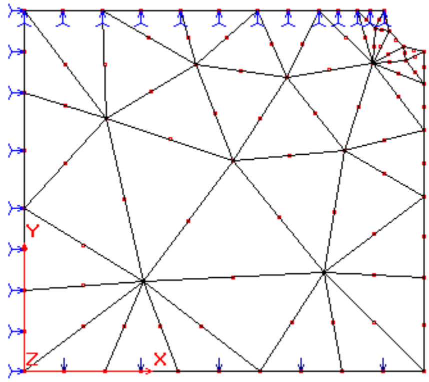
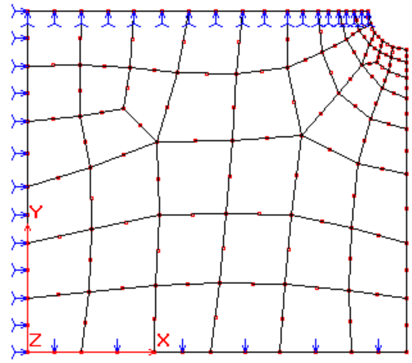


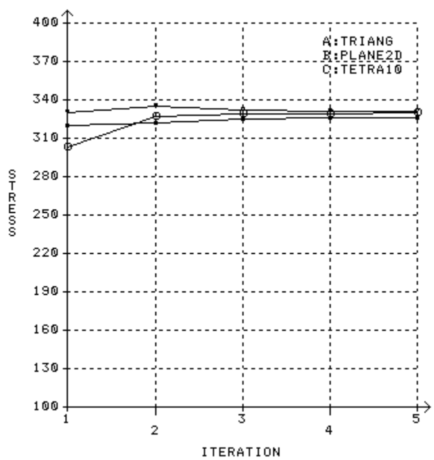
Figure S95-2: Finite Element Model with Different Element Types

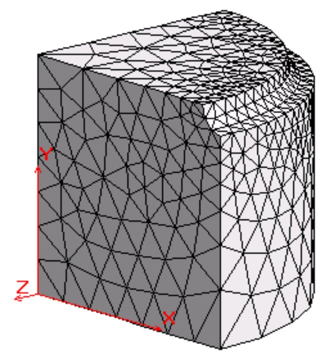


S95A: TRIANG elements



S95B: PLANE2D elements





S95C: TETRA10 elements

Convergence Plots Using Different Element Types

COMPARISON OF RESULTS:

	Theory	COSMOS/M	Relative Error
Max. Stress in Y-Direction (TRIANG)	305 psi	337 psi (p-order = 4)	10.5%
Max. Stress in Y-Direction (PLANE2D)	305 psi	333 psi (p-order = 8)	9.2%
Max. Stress in Y-Direction (TETRA10)	305 psi	339 psi (p-order = 5)	10.5%

REFERENCE:

Walter D. Pilkey, “Formulas For Stress, Strain, and Structural Matrices,” Wiley-Interscience Publication, JohnWiley & Sons, Inc., 1994, pp. 267.

3 Modal (Frequency) Analysis

Introduction

This chapter contains verification problems to demonstrate the accuracy of the Modal Analysis module DSTAR.

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F1: Natural Frequencies of a Two-Mass Spring System

TYPE:

Mode shape and frequency, truss and mass element (TRUSS3D, MASS).

REFERENCES:

Thomson, W. T., "Vibration Theory and Application," Prentice-Hall, Inc., Englewood Cliffs, New Jersey, 2nd printing, 1965, p. 163.

PROBLEM:

Determine the normal modes and natural frequencies of the system shown below for the values of the masses and the springs given.

GIVEN:

$m_2 = 2m_1 = 1$ lb-sec^2/in

$k_2 = k_1 = 200$ lb/in

$k_c = 4k_1 = 800$ lb/in

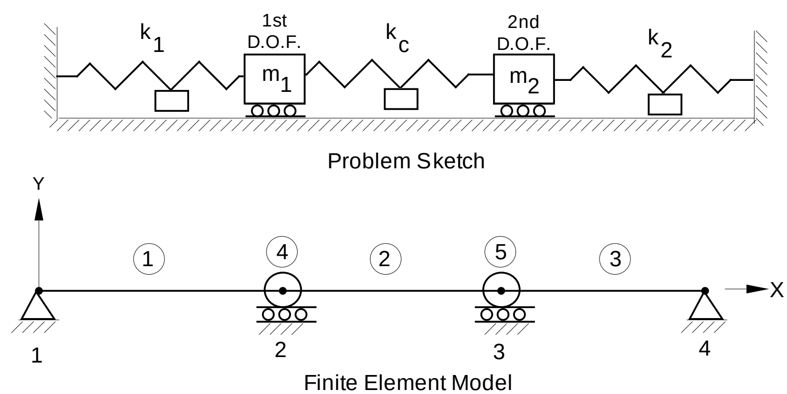
COMPARISON OF RESULTS:

	F_1, Hz	F_2, Hz
Theory	2.581	8.326
COSMOS/M	2.581	8.326

MODELING HINTS:

Truss elements with zero density are used as springs. Two dynamic degrees of freedom are selected at nodes 2 and 3 and masses are input as concentrated masses at nodes 2 and 3.

Figure F1-1



F2: Frequencies of a Cantilever Beam

TYPE:

Mode shape and frequency, plane element (PLANE2D).

REFERENCE:

Flugge, W., “Handbook of Engineering Mechanics,” McGraw-Hill Book Co., Inc., New York, 1962, pp. 61-6, 61-9.

PROBLEM:

Determine the fundamental frequency, f, of the cantilever beam of uniform cross section A.

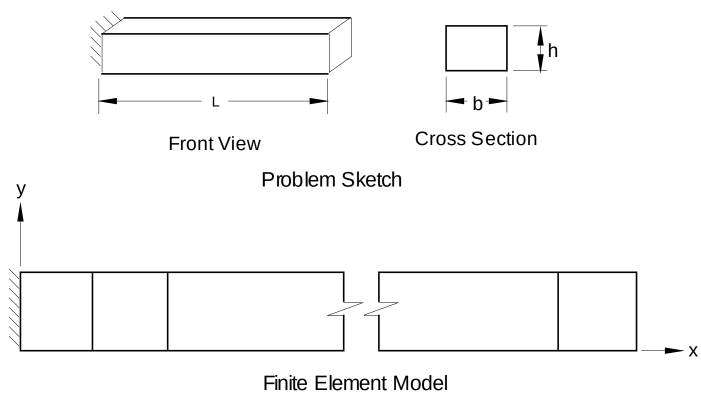
GIVEN:

E = 30×10^6 psi
L = 50 in
h = 0.9 in
b = 0.9 in
A = $0.81\ in^2$
ν = 0
ρ = $0.734E\text{-}3\ lb\ sec^2/in^4$

COMPARISON OF RESULTS

	F_1, Hz	F_2, Hz	F_3, Hz
Theory	11.79	74.47	208.54
COSMOS/M	11.72	73.35	206.68

Figure F2-1



F3: Frequency of a Simply Supported Beam

TYPE:

Mode shapes and frequencies, beam element (BEAM3D).

REFERENCE:

Thomson, W. T., “Vibration Theory and Applications,” Prentice-Hall, Inc., Englewood Cliffs, New Jersey, 2nd printing, 1965, p. 18.

PROBLEM:

Determine the fundamental frequency, f, of the simply supported beam of uniform cross section A.

GIVEN:

E = 30×10^6 psi

L = 80 in

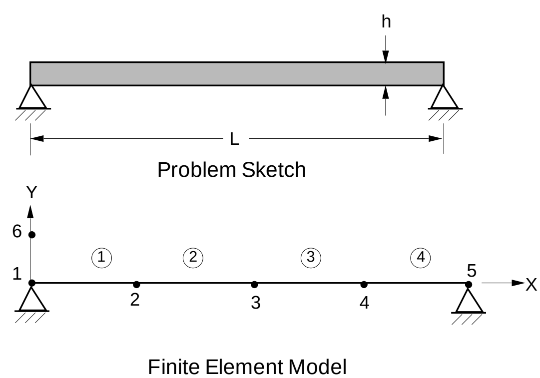
ρ = 0.7272E-3 lb-sec^2/in^4

A = 4 in^2

I = 1.3333 in^4

h = 2 in

Figure F3-1



ANALYTICAL SOLUTION:

$F_i = (i\pi)^2 \, (EI/mL^4)^{(1/2)}$

i = Number of frequencies

COMPARISON OF RESULTS:

	F_1, Hz	F_2, Hz	F_3, Hz
Theory	28.78	115.12	259.0
COSMOS/M	28.78	114.31	242.7

F4: Natural Frequencies of a Cantilever Beam

TYPE:

Mode shapes and frequencies, beam element (BEAM3D).

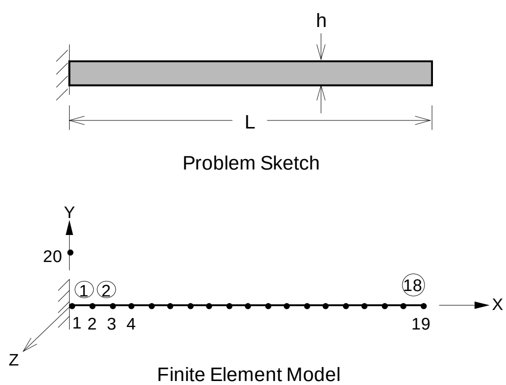
REFERENCE:

Thomson, W. T., “Vibration Theory and Applications,” Prentice-Hall, Inc., Englewood Cliffs, New Jersey, 2nd printing, 1965, p. 278, Ex. 8.5-1, and p. 357.

PROBLEM:

Determine the first three natural frequencies, f, of a uniform beam clamped at one end and free at the other end.

Figure F4-1



GIVEN:

E = 30×10^6 psi

I = 1.3333 in^4

A = 4 in^2

h = 2 in

L = 80 in

ρ = 0.72723E-3 lb sec^2/in^4

COMPARISON OF RESULTS:

	F_1, Hz	F_2, Hz	F_3, Hz
Theory	10.25	64.25	179.9
COSMOS/M	10.24	63.95	178.5

F5: Frequency of a Cantilever Beam with Lumped Mass

TYPE:

Mode shape and frequency, beam and mass elements (BEAM3D, MASS).

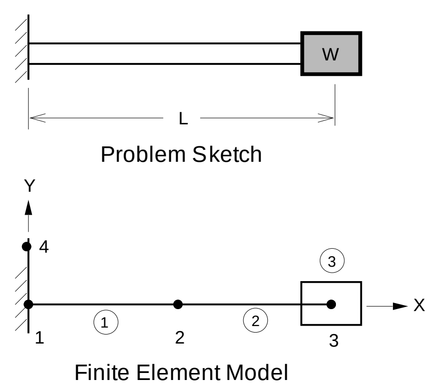
REFERENCE:

William, W. Seto, "Theory and Problems of Mechanical Vibrations," Schaum's Outline Series, McGraw-Hill Book Co., Inc., New York, 1964, p. 7.

PROBLEM:

A steel cantilever beam of length 10 in has a square cross-section of 1/4 x 1/4 inch. A weight of 10 lbs is attached to the free end of the beam as shown in the figure. Determine the natural frequency of the system if the mass is displaced slightly and released.

Figure F5-1



GIVEN:

E = 30×10^6 psi

W = 10 lb

L = 10 in

COMPARISON OF RESULTS:

	F, Hz
Theory	5.355
COSMOS/M	5.359

F6: Dynamic Analysis of a 3D Structure

TYPE:

Mode shapes and frequencies, pipe and mass elements (PIPE, MASS).

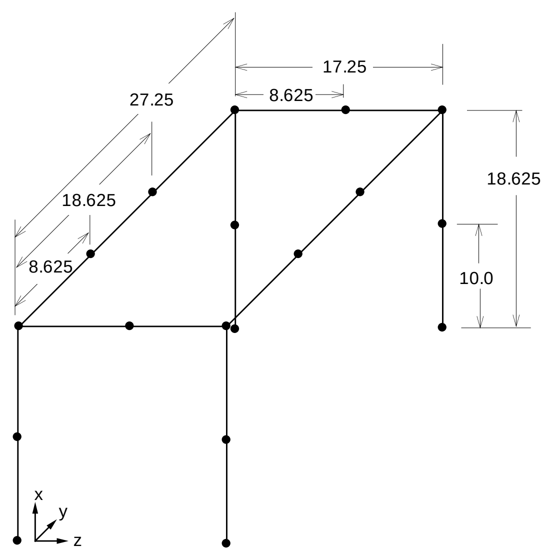
REFERENCE:

"ASME Pressure Vessel and Piping 1972 Computer Programs Verification," Ed. by I. S. Tuba and W. B. Wright, ASME Publication I-24, Problem 1.

PROBLEM:

Find the natural frequencies and mode shapes of the 3D structure given below.

Figure F6-1



Problem Sketch and Finite Element Model

GIVEN:

Each member is a pipe.

Outer diameter = 2.375 in

Thickness = 0.154 in

E = 27.9×10^6 psi

ν = 0.3

The masses are represented solely by lumped masses as shown in the figure.

$M_1 = M_2 = M_4 = M_6 = M_7 = M_8 = M_9 = M_{11} = M_{13} = M_{14} = 0.00894223$ lb sec^2/in

$M_3 = M_5 = M_{10} = M_{12} = 0.0253816$ lb sec^2/in

COMPARISON OF RESULTS:

	F_1, Hz	F_2, Hz	F_3, Hz	F_4, Hz	F_5, Hz
Theory	111.5	115.9	137.6	218.0	404.2
COSMOS/M	111.2	115.8	137.1	215.7	404.2

F7A, F7B: Dynamic Analysis of a Simply Supported Plate

TYPE:

Mode shapes and frequencies, shell elements (SHELL4 and SHELL6).

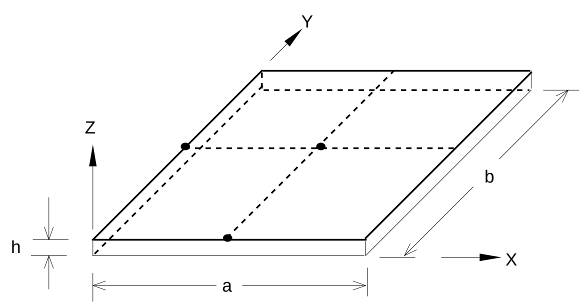
REFERENCE:

Leissa, A.W. “Vibration of Plates,” NASA, sp-160, p. 44.

PROBLEM:

Obtain the first natural frequency for a simply supported plate.

Figure F7-1



Problem Sketch and Finite Element Model

GIVEN:

E = 30,000 kips

ν = 0.3

h = 1 in

a = b = 40 in

ρ = 0.003 kips sec^2/in^4

NOTE:

Due to double symmetry in geometry and the required mode shape, a quarter of the plate is taken for modeling.

COMPARISON OF RESULTS

The first natural frequency of the plate is 5.94 Hz.

	F7A: SHELL4	F7B: SHELL6 (Curved)	F7B: SHELL6 (Assembled)
COSMOS/M	5.93 Hz	5.94 Hz	5.93

F8: Clamped Circular Plate

TYPE:

Mode shapes and frequencies, thick shell element (SHELL3T).

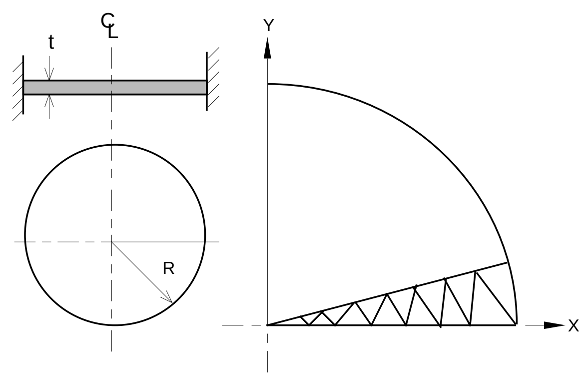
REFERENCE:

Leissa A.W., “Vibration of Plates,” NASA sp-160, p. 8.

PROBLEM:

Obtain the first three natural frequencies.

Figure F8-1



Problem Sketch

Finite Element Model

GIVEN:

E = 30 x 10^6 psi

ν = 0.3

ρ = 0.00073 (lb/in^4) sec^2

R = 40 in

t = 1 in

NOTE:

Since a quarter of the plate is used for modeling, the second natural frequency is not symmetric (s = 0, n = 1) and will not be calculated. This is an example to show that symmetry should be used carefully.

COMPARISON OF RESULTS:

Frequency No.	s*	n*	Theory (Hz)	COSMOS/M (Hz)
1	0	0	62.30	62.40
2	0	2	212.60	212.53
3	1	0	242.75	240.30

s* refers to the number of nodal circles

n***** refers to the number of nodal diameters

F9: Frequencies of a Cylindrical Shell

TYPE:

Mode shapes and frequencies, shell element (SHELL4).

REFERENCE:

Kraus, “Thin Elastic Shells,” John Wiley & Sons, Inc., p. 307.

PROBLEM:

Determining the first three natural frequencies.

GIVEN:

E = 30×10^6 psi

ν = 0.3

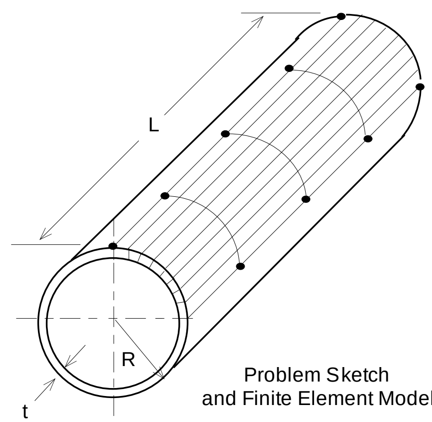
ρ = 0.00073 $(\text{lb-sec}^2)/\text{in}^4$

L = 12 in

R = 3 in

t = 0.01 in

Figure F9-1



Problem Sketch
and Finite Element Model

NOTE:

Due to symmetry in geometry and the mode shapes of the first three natural frequencies, 1/8 of the cylinder is considered for modeling.

COMPARISON OF RESULTS:

	F_1, Hz	F_2, Hz	F_3, Hz
Theory	552	736	783
COSMOS/M	553.69	718.50	795.60

F10: Symmetric Modes and Natural Frequencies of a Ring

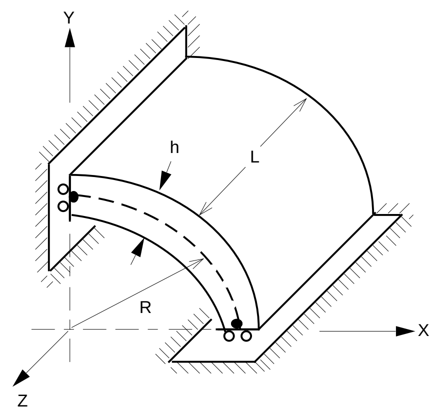
TYPE:

Mode shapes and frequencies, shell element (SHELL4).

REFERENCE:

Flugge, W. “Handbook of Engineering Mechanics,” First Edition, McGraw-Hill, New York, p. 61-19.

Figure F10-1



Problem Sketch

PROBLEM:

Determine the first two natural frequencies of a uniform ring in symmetric case.

GIVEN:

E = 30E6 psi

ν = 0

L = 4 in

h = 1 in

R = 1 in

ρ = 0.25E-2 (lb sec^2)/in^4

COMPARISON OF RESULTS:

	F_1, Hz	F_2, Hz
Theory	135.05	735.14
COSMOS/M	134.92	723.94

F11A, F11B: Eigenvalues of a Triangular Wing

TYPE:

Mode shapes and frequencies, triangular shell elements (SHELL3 and SHELL6).

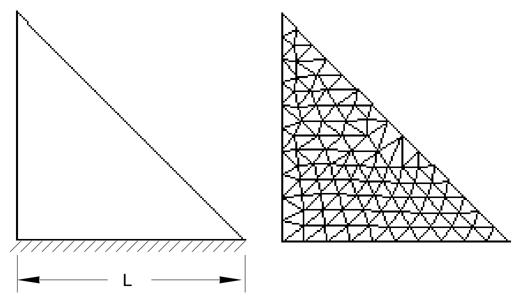
REFERENCE:

"ASME Pressure Vessel and Piping 1972 Computer Programs Verification," ed. by I. S. Tuba and W. B. Wright, ASME Publication I-24, Problem 2.

PROBLEM:

Calculate the natural frequencies of a triangular wing as shown in the figure.

Figure F11-1



Problem Geometry   Finite Element Model

GIVEN:

E = 6.5×10^6 psi

ν = 0.3541

ρ = 0.166E-3 lb sec^2/in^4

L = 6 in

Thickness = 0.034 in

COMPARISON OF RESULTS:

Natural Frequencies (Hz):

Frequency No.	Reference	COSMOS/M		
		SHELL3	SHELL6 (Curved)	SHELL6 (Assembled)
1	55.9	55.8	56.137	55.898
2	210.9	206.5	212.708	210.225
3	293.5	285.5	299.303	291.407

F12: Vibration of an Unsupported Beam

TYPE:

Mode shapes and frequencies, rigid body modes, beam element (BEAM3D).

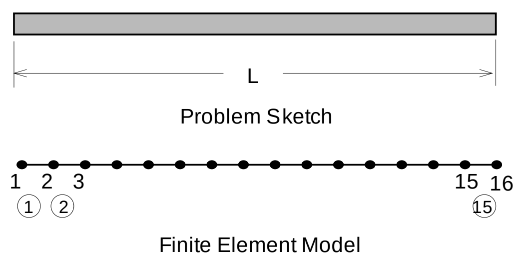
REFERENCE:

Timoshenko, S. P., Young, O. H., and Weaver, W., “Vibration Problems in Engineering,” 4th ed., John Wiley and Sons, New York, 1974, pp. 424-425.

PROBLEM:

Determine the elastic and rigid body modes of vibration of the unsupported beam shown below.

Figure F12-1



GIVEN:

L = 100 in

E = 1×10^8 psi

r = 0.1 in

ρ = 0.2588E-3 lb sec^2/in^4

ANALYTICAL SOLUTION:

The theoretical solution is given by the roots of the equation Cos KL Cosh KL = 1 and the frequencies are given by:

$f_i = K_i^2 (EI/\rho A)^{(1/2)}/(2\pi)$

i = Number of natural frequencies

$K_i = (i + 0.5)\pi/L$

A = area of cross-section

ρ = Mass Density

COMPARISON OF RESULTS:

	Mode 1	Mode 2	Mode 3	Mode 4	Mode 5	Mode 6
Theory F, Hz	0	0	11.07	30.51	59.81	98.86
Theory (ki)	(0)	(0)	(4.73)	(7.853)	(10.996)	(14.137)
COSMOS/M F, Hz	0	0	10.92	29.82	57.94	94.94

NOTE:

First two modes are rigid body modes.

F13: Frequencies of a Solid Cantilever Beam

TYPE:

Mode shapes and frequencies, hexahedral solid element (SOLID).

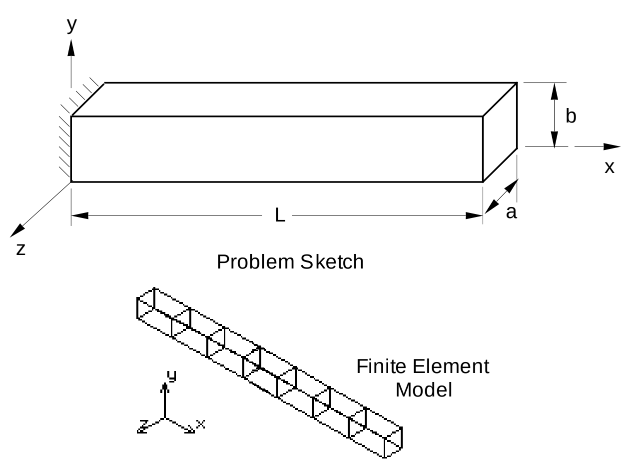
REFERENCE:

Thomson, W. T., “Vibration Theory and Applications,” Prentice-Hall, Inc., Englewood Cliffs, N. J., 2nd printing, 1965, p.275, Ex. 8.5-1, and p. 357.

PROBLEM:

Determine the first three natural frequencies of a uniform beam clamped at one end and free at the other end.

Figure F13-1



GIVEN:

E = 30 x 10^6 psi

a = 2 in

b = 2 in

L = 80 in

ρ = 0.00072723 lb-sec^2/in^4

COMPARISON OF RESULTS:

	F_1, Hz	F_2, Hz	F_3, Hz
Theory	10.25	64.25	179.91
COSMOS/M	10.24	63.95	178.38

F14: Natural Frequency of Fluid

TYPE:

Mode shapes and frequencies, truss elements (TRUSS2D).

REFERENCE:

William, W. Seto, “Theory and Problems of Mechanical Vibrations,” Schaum’s Outline Series, McGraw-Hill Book Co., Inc., New York, 1964, p. 7.

PROBLEM:

A manometer used in a fluid mechanics laboratory has a uniform bore of cross-section area A. If a column of liquid of length L and weight density ρ is set into motion, as shown in the figure, find the frequency of the resulting motion.

GIVEN:

A = 1 in^2
ρ = 9.614E-5 lb sec^2/in^4
L = 51.4159 in
E = 1E5 psi

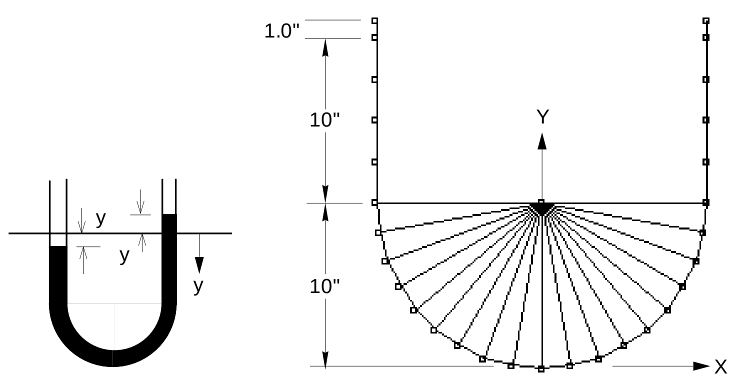
COMPARISON OF RESULTS:

	F, Hz
Theory	0.617
COSMOS/M	0.617

NOTE:

The mass of fluid is lumped at nodes 2 to 28. The boundary elements are applied at nodes 6 to 24.

Figure F14-1



Problem Sketch

Finite Element Model

F16A, F16B: Vibration of a Clamped Wedge

TYPE:

Mode shapes and frequencies, thick shell elements (SHELL3T, SHELL4T).

REFERENCE:

Timoshenko, S., and Young, D. H., “Vibration Problems in Engineering,” 3rd Edition, D. Van Nostrand Co., Inc., New York, 1955, p. 392.

PROBLEM:

Determine the fundamental frequency of lateral vibration of a wedge shaped plate. The plate is of uniform thickness t, base 3b, and length L.

GIVEN:

E = 30×10^6 psi

ρ = 7.28E-4 lb sec^2/in^4

t = 1 in

b = 2 in

L = 16 in

MODELING HINTS:

Only in-plane (in x-y plane) frequencies along y-direction are considered. In order to find better results, out-of-plane displacements (z-direction) are restricted.

The effect of different elements and meshes is also considered.

ANALYTICAL SOLUTION:

The first in-plane natural frequency calculated by:

$$f_1 = \frac{5.315\,b}{2\,\pi L^2}\sqrt{\frac{E}{3\rho}}$$

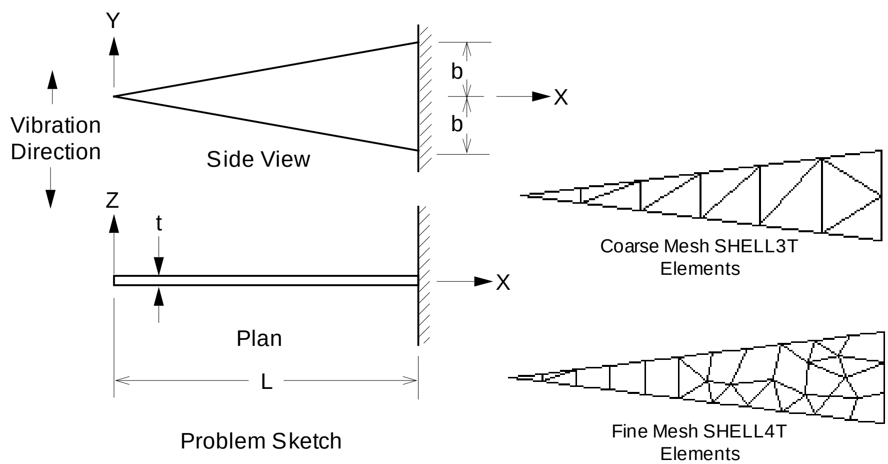
Using approximate RITZ method, for first and second natural frequencies:

$$f_1 = \frac{5.319\,b}{2\,\pi L^2}\sqrt{\frac{E}{3\rho}} \qquad f_2 = \frac{17.301\,b}{2\,\pi L^2}\sqrt{\frac{E}{3\rho}}$$

COMPARISON OF RESULTS:

	Natural Frequency (Hz)	
	First	**Second**
Reference		
Exact	774.547	---
Ritz	775.130	2521.265
COSMOS/M		
SHELL3T (F16A)	813.45	2280.78
SHELL4T (F16B)	789.12	2309.54

Figure F16A-1



F17: Lateral Vibration of an Axially Loaded Bar

TYPE:

Mode shapes and frequencies, in-plane effects, beam elements (BEAM3D).

REFERENCE:

Timoshenko, S., and Young, D. H., “Vibration Problems in Engineering,” 3rd Edition, D. Van Nostrand Co., Inc., New York, 1955, p. 374.

PROBLEM:

Determine the fundamental frequency of lateral vibration of a wedge shaped plate. The plate is of uniform thickness t, base 3b, and length L.

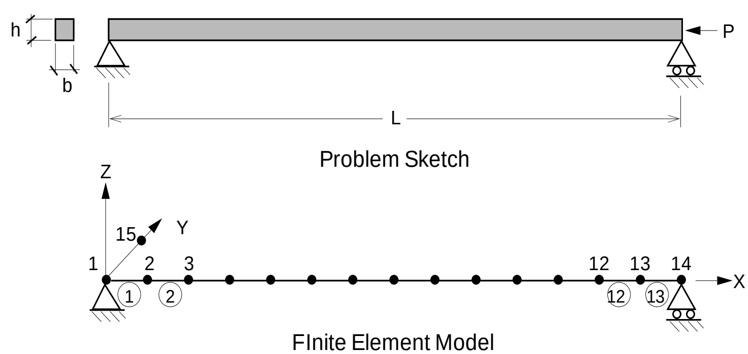
GIVEN:

E	= 30E6 psi	b	= h = 2 in
ρ	= 7.2792E-4 lb sec^2/in^4	L	= 80 in
g	= 386 in/sec^2	P	= 40,000 lb

COMPARISON OF RESULTS:

	F_1, Hz	F_2, Hz	F_3, Hz
Theory	17.055	105.32	249.39
COSMOS/M	17.055	105.32	249.34

Figure F17-1



F18: Simply Supported Rectangular Plate

TYPE:

Mode shapes and frequencies, in-plane effects, shell element (SHELL4).

REFERENCE:

Leissa, A.W., “Vibration of Plates,” NASA, p-160, p. 277.

PROBLEM:

Obtain the fundamental frequency of a simply supported plate with the effect of in-plane forces. Nx = 33.89 lb/in applied at x = 0 and x = a.

GIVEN:

$E = 30,000$ psi
$\nu = 0.3$
$h = 1$ in
$a = b = 40$ in
$\rho = 0.0003$ (lb sec^2)/in^4
$P = 33.89$ psi

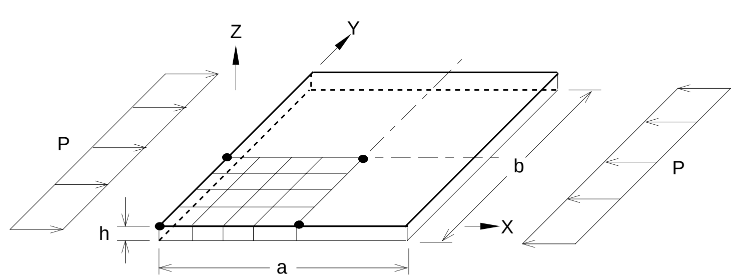
COMPARISON OF RESULTS:

	F, Hz
Theory	4.20
COSMOS/M	4.19

NOTE:

Due to double symmetry in geometry, loads and the mode shape, a quarter plate is taken for modeling.

Figure F18-1



Problem Sketch and Finite Element Model

F19: Lowest Frequencies of Clamped Cylindrical Shell for Harmonic No. = 6

TYPE:

Mode shapes and frequencies, axisymmetric shell elements (SHELLAX).

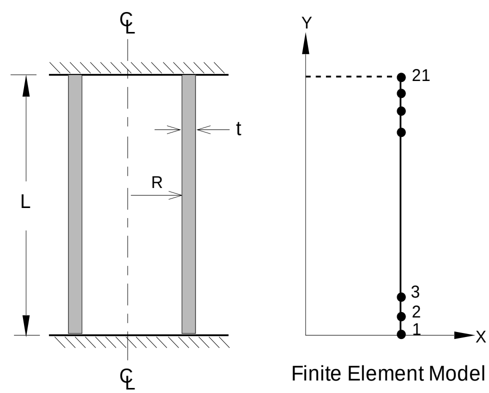
REFERENCE:

Leissa, A. W., “Vibration of Shells,” NASA sp-288, p. 92-93 (1973).

PROBLEM:

To find the lowest natural frequency of vibration for the cylinder fixed at both ends.

Figure F19-1



Finite Element Model

Problem Sketch

GIVEN:

R = 3 in
L = 12 in
t = 0.01 in
E = 30 x 10^6 psi
ν = 0.35
ρ = 0.000730 lb sec^2/in^4
Range of circumferential harmonics (n) = 4 to 7

MODELING HINTS:

All the 21 nodes are spaced equally along the meridian of cylinder. The number of circumferential harmonics (lobes) for each frequency analysis is to be specified and lowest frequency is sought.

COMPARISON OF RESULTS:

Harmonic No. (n)	First Frequency (Hz)		
	Theory	Experiment	COSMOS/M
(4) *	926	700	777.45
(5) *	646	522	592.6
(6) *	563	525	549.4
(7) *	606	592	609.7

* You need to re-execute the analysis by specifying these harmonic numbers under the A_FREQUENCY command. The lowest natural frequency is 549.6 Hz corresponding to harmonic number = 6.

F20A, F20B, F20C, F20D, F20E, F20F, F20G, F20H: Dynamic Analysis of Cantilever Beam

TYPE:

Mode shapes and frequencies, multifield elements, 4- and 8-node PLANE2D, SHELL4T, 6-node TRIANG, TETRA10, 8- and 20-node SOLID, TETRA4R, and SHELL6.

PROBLEM:

Compare the first two natural frequencies of a cantilever beam modeled by each of the above element types.

GIVEN:

E = 10^7 psi

ρ = 245×10^{-3} lb-sec^2/in^4

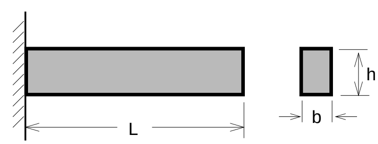
b = 0.1 in

h = 0.2 in

L = 6 in

n = 0.3

Figure F20-1



Problem Sketch

COMPARISON OF RESULTS:

The theoretical solutions for the first and second mode are: 181.17 and 1136.29 Hz.

Input File	Element	1st Mode	Error (%)	2nd Mode	Error (%)
F20A	PLANE2D 4-node	180.71	0.2	1127.96	0.7
F20B	PLANE2D 8-node	181.15	0.0	1153.52	1.53
F20C	TRIANG 6-node	183.35	1.2	1182.90	4.1
F20D	TETRA10	183.10	1.0	1184.85	4.3
F20E	SOLID 8-node	181.64	0.2	1134.67	0.2
F20F	SOLID 20-node	179.72	0.8	1111.16	2.2
F20G	TETRA4R	190.24	5.1	1182.72	4.1
F20H	SHELL6 (Curved)	183.371	1.2	1182.87	4.1
	SHELL6 (Assembled)	183.357	1.2	1182.54	4.1

F21: Frequency Analysis of a Right Circular Canal of Fluid with Variable Depth

TYPE:

Mode shapes and frequencies, fluid sloshing, plane strain elements (PLANE2D).

REFERENCE:

Budiansky, B., “Sloshing of Liquids in Circular Canals and Spherical Tank,” J. Aerospace Sci, 27, p. 161-173, (1960).

PROBLEM:

A right circular canal with radius R is half-filled by an incompressible liquid (see Figure F21-1). Determine the first two natural frequencies with mode shapes antisymmetric about the Y-axis.

GIVEN

R = 56.4 in

H/R = 0

ρ = 0.9345E-4 lb sec^2/in^4

EX = 3E5 lb/in^2

Where:

EX = bulk modulus

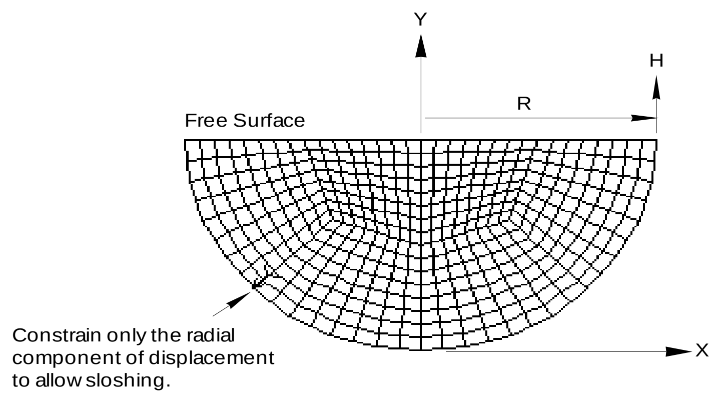
NOTES:

1. A small shear modulus SXY = EX is used to prevent numerical instability.
2. The radial component of displacements (in local cylindrical coordinate system) is constrained at the curved boundary in order to allow sloshing.
3. The acceleration due to gravity (ACEL command) in the negative direction.
4. PLANE2D plane strain elements are used to solve the current problem since the mode shapes are independent of Z-direction coordinates.
5. For non rectangular geometries, one can expect to obtain some natural frequencies with no significant changes in the free surface profile. This situation is analogous to the rigid modes of a solid structure. Therefore, a negative shift of ω^2 is recommended to prevent this type of sloshing modes.

COMPARISON OF RESULTS:

Mode Number	Analytical Solution (Hz)	COSMOS/M (Hz)
1	0.4858	0.4875
2	Not Available	0.7269
3	0.9031	0.8976

Figure F21-1



Finite Element Model

F22: Frequency Analysis of a Rectangular Tank of Fluid with Variable Depth

TYPE:

Mode shapes and frequencies, fluid sloshing, hexahedral solid (SOLID).

REFERENCE:

Lamb, H., “Hydrodynamics,” 6th edition, Dover Publications, Inc., New York, 1945.

PROBLEM:

A rectangular tank with dimensions A and B in X- and Z-directions is partially filled by an incompressible liquid (see Figure F22-1). Determine the first two natural frequencies.

GIVEN:

A = 48 in

B = 48 in

H = 20 in

ρ = 0.9345E-4 lb sec^2/in^4

EX = 3E5 lb/in^2

Where:

EX = bulk modulus

NOTE:

Please refer to notes (1), (2), (3), (4) and (5) in Problem F21.

COMPARISON OF RESULTS:

The analytical solution for natural frequencies is as follows:

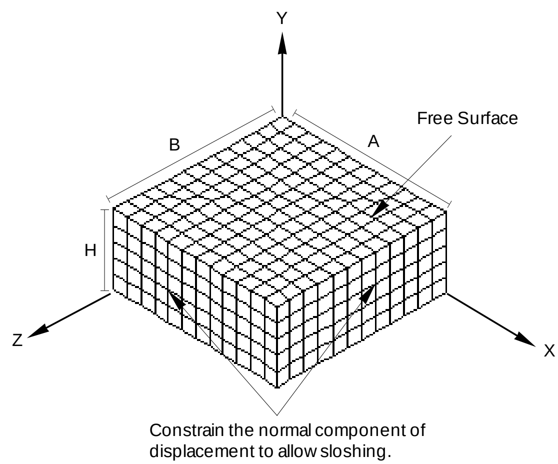
$$b_{ij} = \frac{1}{2}\sqrt{\frac{g}{\pi}\left[\frac{i^2}{a^2}+\frac{j^2}{b^2}\right]\tanh \pi h\left(\frac{i^2}{a^2}+\frac{j^2}{b^2}\right)^{\frac{1}{2}}}\ \text{Hz}$$

where i and j represent the order number in X- and Z-directions respectively.

The comparison of analytical solutions with those obtained using COSMOS/M for various values of i and j are tabulated below.

Frequency Number I / J	Analytical Solution (Hz)	COSMOS/M (Hz)
1 / 0	—	—
0 /1	0.7440	0.7422
1 / 2	0.9286	0.9199

Figure F22-1



Finite Element Model

F23: Natural Frequency of Fluid in a Manometer

TYPE:

Mode shapes and frequencies, fluid sloshing, plane strain elements (PLANE2D).

REFERENCE:

William, W. Seto, “Theory and Problems of Mechanical Vibrations,” Schaum's Outline Series, McGraw-Hill Book Co., Inc., New York, 1964, p. 7.

PROBLEM:

A manometer used in a fluid mechanics laboratory has a uniform bore of cross-sectional area A. If a column of liquid of length L and weight density r is set into motion as shown in the figures, find the frequency of the resulting motion.

GIVEN:

A = 0.5 in^2

ρ = 0.9345E-4 lb sec^2/in^4

L = 26.4934 in (length of fluid in the manometer)

EX = 3E5 lb/in^2

Where:

EX = bulk modulus

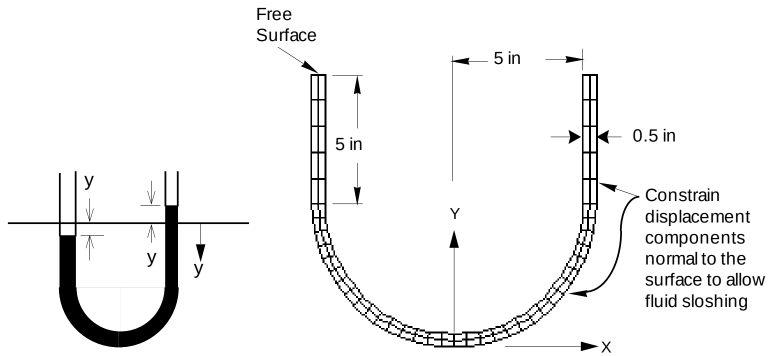
COMPARISON OF RESULTS:

	F, Hz
Analytical Solution *	0.8596
COSMOS/M	0.8623

NOTE:

A small shear modulus GXY = EX(1.0E-9) is used to prevent numerical instability. Global and local constraints are applied normal to the boundary to prevent leaking of the fluid. Acceleration due to gravity (ACEL command) in the negative y-direction should be included for problems with free surfaces

Figure F23-1



Problem Sketch

Finite Element Model

F24: Modal Analysis of a Piezoelectric Cantilever

TYPE:

Mode shapes and frequencies using solid piezoelectric element (SOLIDPZ).

REFERENCE:

J. Zelenka, "Piezoelectric Resonators and their Applications", Elsevier Science Publishing Co., Inc., New York, 1986.

PROBLEM:

A piezoelectric transducer with a polarization direction along its longitudinal direction has electrodes at two ends. Both electrodes are grounded to represent a short-circuit condition. All non-prescribed voltage D.O.F.'s are condensed out after assemblage of stiffness matrix. In this problem, the longitudinal mode of vibration is under consideration.

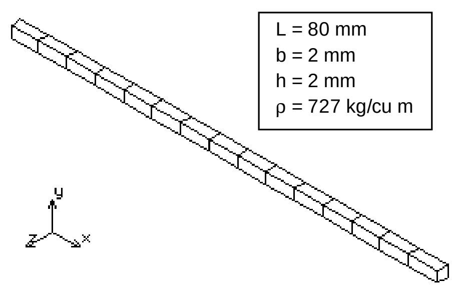
GIVEN:

L = 80 mm

b = h = 2 mm

Density = 727 Kg/m^3

Figure F24-1



NOTE:

To constrain voltage degrees of freedom for piezoelectric application, use the RX component of displacement in the applicable constraint commands (DND, DCR, DSF, etc.). There is no rotational degree of freedom for SOLID elements in COSMOS/M.

COMPARISON OF RESULTS:

For the sixth mode of vibration in this problem (longitudinal mode):

	Sixth Mode of Vibration (Longitudinal Mode)
Theory	690 Hz
COSMOS/M	685 Hz

F25: Frequency Analysis of a Stretched Circular Membrane

TYPE:

Frequency analysis using the nonaxisymmetric mode shape option (SHELLAX).

REFERENCE:

Leissa, A. W., "Vibration of Shells," NASA-P-SP-288, 1973.

PROBLEM:

Find the first three frequencies of a stretched circular membrane.

GIVEN:

R = 15 in
E = 30E6 psi
T = 100 lb/in
t = 0.01 in (Thickness)
ρ = 0.0073 lb-sec^2/in^4

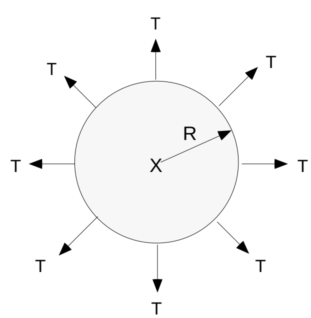
COMPARISON OF RESULTS

Natural Frequency No.	Theory (Hz)	COSMOS/M (Hz)	Error (%)
1	94.406	93.73	0.72
2	216.77	212.95	1.76
3	339.85	329.76	2.97

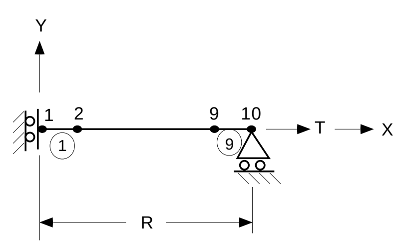
MODELING HINTS:

A total of 9 elements are considered as shown. The stretching load of 1500 lb for a one radian section of the shell is applied with the inplane loading flag turned on for frequency calculations. All frequencies are found for circumferential harmonic number 0.

Figure F25-1



Problem Sketch



Finite Element Model

F26: Frequency Analysis of a Spherical Shell

TYPE:

Frequency analysis using the nonaxisymmetric mode shape option (SHELLAX).

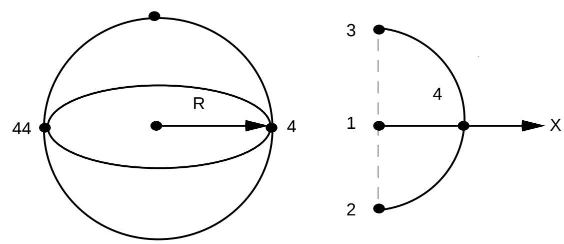
REFERENCE:

Krause, H., “Thin Elastic Shells,” John Wiley, Inc., New York, 1967.

PROBLEM:

Find the first eight frequencies of the spherical shell shown here for the circumferential harmonic number 2.

Figure F26-1



Problem Sketch   Finite Element Model

GIVEN:

R = 10 in

E = 1E7 psi

ρ = 0.0005208 lb-sec^2/in^4

ν (NUXY) = 0.3

t (Thickness) = 0.1 in

COMPARISON OF RESULTS:

Natural Frequency No.	Theory (Hz)	COSMOS/M (Hz)	Error (%)
1	1620	1622	0.12
2	1919	1923	0.20
3	2035	2044	0.44
4	2093	2110	0.81
5	2125	2153	1.32
6	2145	2188	2.00
7	2159	2224	3.01
8	2168	2262	4.34

F27A, F27B: Natural Frequencies of a Simply-Supported Square Plate

TYPE:

Frequency analysis, Guyan reduction, SHELL4 elements.

Case A: Guyan Reduction

Case B: Consistent Mass

PROBLEM:

Natural frequencies of a simply-supported plate are calculated. Utilizing the symmetry of the model, only one quarter of the plate is modeled and the first three symmetric modes of vibration are calculated. The mass is lumped uniformly at master degrees of freedom.

GIVEN:

$L = 30$ in

$h = 0.1$ in

$\rho = 8.29 \times 10^{-4}$ $(\text{lb sec}^2)/\text{in}^4$

$\nu = 0.3$

$E = 30.E6$ psi

ANALYTICAL SOLUTION:

Theoretical results can be obtained from the equation:

$$\omega_{mn} = r^2 D / L^2 U * (m^2 + n^2)$$

Where:

$$D = Eh^3 / 12(1 - \nu^2)$$

$$U = \rho h$$

COMPARISON OF RESULTS:

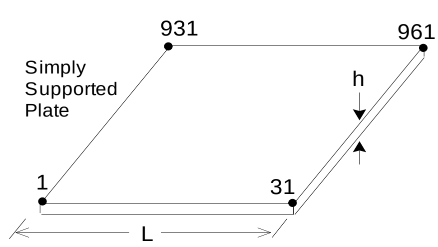
Normalized mode shape displacements for the nodes connected by the rigid bar.

	Natural Frequency (Hz)		
	First	Second	Third
Theory	5.02	25.12	25.12
Case A: Guyan Reduction	5.03	25.15	25.20
Case B: Consistent Mass	5.02	25.11	25.11

Total Mass = $\rho * \nu = 8.29 * 10^{-4} * 0.1 * 30 * 30 = .07461$

Lumped Mass at Master Nodes = .07461/64 = 1.16E-3

Figure F27-1



Problem Sketch

F28: Cylindrical Roof Shell

TYPE:

Natural mode shape and frequency, shell and rigid bar elements.

PROBLEM:

Determine the first frequency and mode shape of the shell roof shown below.

GIVEN:

r = 25 ft

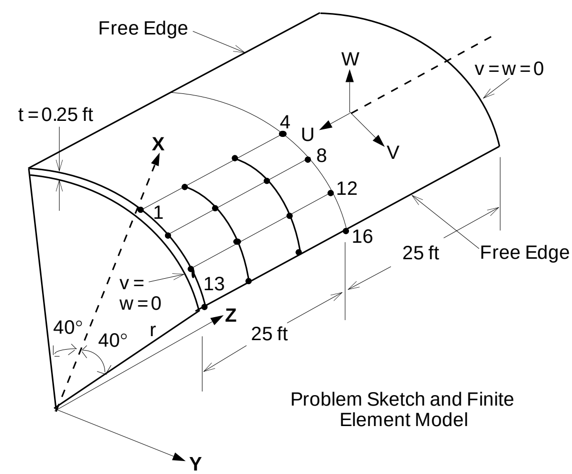
E = 4.32E12,
4.32E11, and
4.32E10 psi

ν = 0

MODELING HINTS:

Due to symmetry, a quarter of the shell roof is considered in the modeling. Nodes 8 and 12 are connected by a rigid bar.

Figure F28-1



Problem Sketch and Finite Element Model

COMPARISON OF RESULTS:

Normalized mode shape displacements for the nodes connected by the rigid bar.

Method	Young's Modulus	Z-Rotation		R8 / R12
		Node 8 (R8)	Node 12 (R12)	
Theory COSMOS/M	4.32E12	-0.5642901E-2 -0.5720E-2	-0.5642901E-2 -0.5720E-2	1.000 1.000
Theory COSMOS/M	4.32E11	-0.5654460E-2 -0.5720E-2	-0.5654460E-2 -0.5720E-2	1.000 1.000
Theory COSMOS/M	4.32E10	-0.5693621E-2 -0.5720E-2	-0.5693621E-2 -0.5720E-2	1.000 1.000

F29A, B, C: Frequency Analysis of a Spinning Blade

TYPE:

Frequency analysis using the Spin Softening and Stress Stiffening Options.

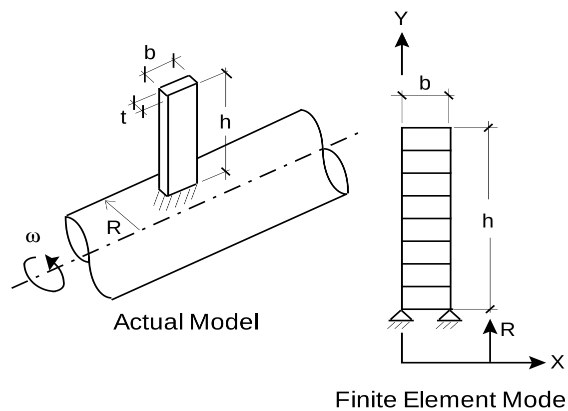
REFERENCE:

W. Carnegie, “Vibrations of Rotating Cantilever Blade,” J. of Mechanical Engineering Science, Vol. 1, No. 3, 1959.

PROBLEM:

Find the fundamental frequency of vibration of a blade cantilevered from a rigid spinning rod.

Figure F29-1



MODELING HINTS:

The blade is cantilevered to a rigid rod. Therefore, the blade may be modeled with a fixed displacement boundary condition at the connection to the rod. The Stress Stiffening effect due to centrifugal load is considered in this model by activating the centrifugal force option in **A_STATIC** command together with the Inplane Loading Flag in **A_FREQUENCY** command.

GIVEN:

R = 150 mm E = 217×10^9 Pa

h = 328 mm ρ = 7850 Kg/m^3

b = 28 mm γ = 0.3

t = 3 mm ω= 314.159 rad/sec

COMPARISON OF RESULTS:

		Fundamental Frequency (Hz)	Error (%)
Theory		52.75	
A	Stress stiffening with spin softening	51.17	3.0
B	Stress stiffening with no spin softening	71.54	36.0
C	No stress stiffening and no spin softening	23.80	54.9

4 Buckling Analysis

Introduction

This chapter contains verification problems to demonstrate the accuracy of the Buckling Analysis module DSTAR.

List of Buckling Verification Problems	
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B2: Instability of Columns	4-3
B3: Instability of Columns	4-4
B4: Simply Supported Rectangular Plate	4-5
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B11: Buckling of a Stepped Column	4-13
B12: Buckling Analysis of a Simply Supported Composite Plate	4-14
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B1: Instability of Columns

TYPE:

Buckling analysis, beam element (BEAM3D).

REFERENCE:

Brush, D. O., and Almroth, B. O., “Buckling of Bars, Plates, and Shells,” McGraw-Hill, Inc., New York, 1975, p. 22.

PROBLEM:

Find the buckling load and deflection mode for a simply supported column.

GIVEN:

E = 30×10^6 psi
h = 1 in
L = 50 in
I = $1/12$ in^4

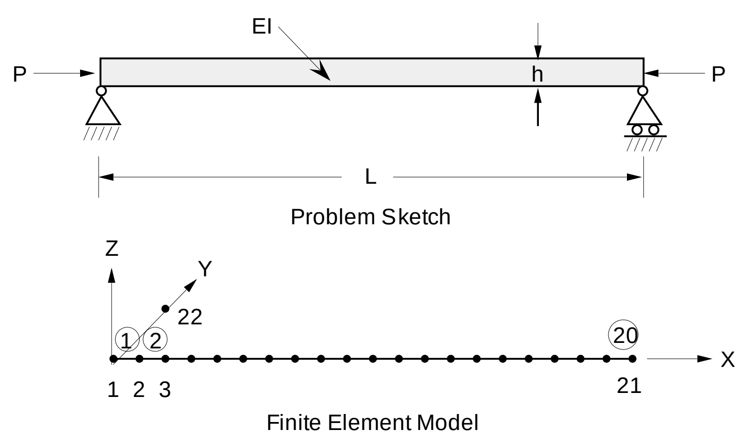
COMPARISON OF RESULTS:

	Theory	COSMOS/M
P_{cr}	9869.6 lb	9869.6 lb

ANALYTICAL SOLUTION:

$$P_{cr} = \pi^2 EI / L^2 = 9869.6 \text{ lb}$$

Figure B1-1



B2: Instability of Columns

TYPE:

Buckling analysis, beam element (BEAM3D).

REFERENCE:

Brush, D. O., and Almroth, B. O., "Buckling of Bars, Plates, and Shells," McGraw-Hill, Inc., New York, 1975, p. 22.

PROBLEM:

Find the buckling load and deflection mode for a clamped-clamped column.

GIVEN:

E = 30×10^6 psi
h = 1 in
L = 50 in
I = $1/12\ in^4$

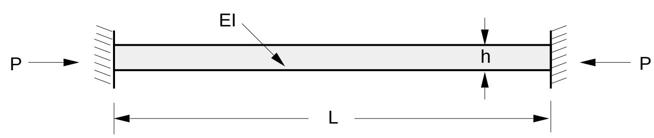
COMPARISON OF RESULTS:

	Theory	COSMOS/M
P_{cr}	39478.4 lb	39478.8 lb

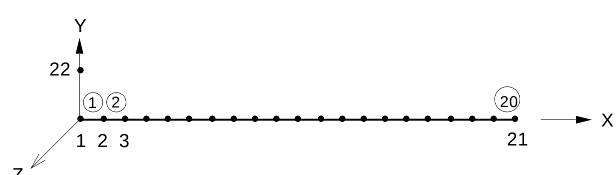
ANALYTICAL SOLUTION:

$$P_{cr} = 4\pi^2 EI / L^2 = 39478.4 \text{ lb}$$

Figure B2-1



Problem Sketch



Finite Element Model

B3: Instability of Columns

TYPE:

Buckling analysis, beam element (BEAM3D).

REFERENCE:

Brush, D. O., and Almroth, B. O., "Buckling of Bars, Plates, and Shells," McGraw-Hill, Inc., New York, 1975, p. 22.

PROBLEM:

Find the buckling load and deflection mode for a clamped-free column.

GIVEN:

E = 30×10^6 psi
h = 1 in
L = 50 in
I = $1/12$ in^4

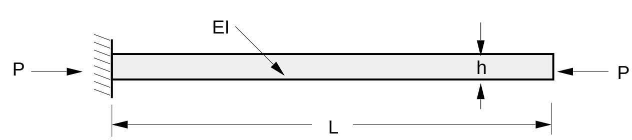
COMPARISON OF RESULTS:

	Theory	COSMOS/M
P_{cr}	2467.4 lb	2467.4 lb

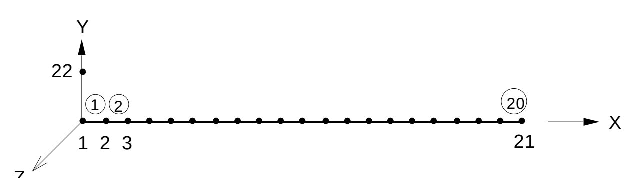
ANALYTICAL SOLUTION:

$$P_{cr} = \pi^2 EI / (4L^2) = 2467.4 \text{ lb}$$

Figure B3-1



Problem Sketch



Finite Element Model

B4: Simply Supported Rectangular Plate

TYPE:

Buckling analysis, shell element (SHELL4).

REFERENCE:

Timoshenko, and Woinosky-Krieger, “Theory of Plates and Shells,” McGraw-Hill Book Co., New York, 2nd Edition, p. 389.

PROBLEM:

Find the buckling load of a simply supported isotropic plate subjected to inplane uniform load p applied at x = 0 and x = a.

GIVEN:

E = 30,000 psi

ν = 0.3

h = 1 in

a = b = 40 in

p = 1 lb/in

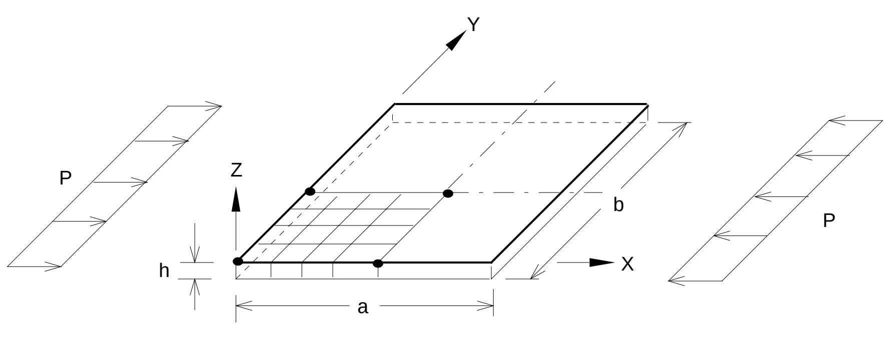
COMPARISON OF RESULTS:

	Theory	COSMOS/M
P_{cr}	67.78 lb	67.85 lb

NOTE:

Due to double symmetry in geometry and loads, a quarter of the plate is taken for modeling.

Figure B4-1



Problem Sketch and Finite Element Model

B5A, B5B: Instability of a Ring

TYPE:

Buckling analysis, shell element (SHELL3, SHELL6).

REFERENCE:

Brush, D. O., and Almroth, B. O., “Buckling of Bars, Plates, and Shells,” McGraw-Hill, Inc., New York, 1975, p. 139.

PROBLEM:

Find the buckling load and deflection mode of a ring under pressure loading.

GIVEN:

E = 10×10^6 psi
R = 5 in
h = 0.1 in
b = 1 in
I = $0.001/12$ in^4

COMPARISON OF RESULTS:

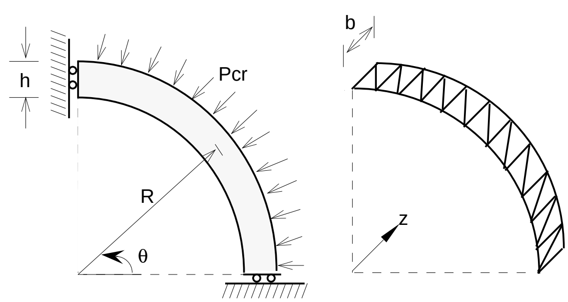
	Theory	COSMOS/M		
		SHELL3	SHELL6 (Curved)	SHELL6 (Assembled)
Pcr	26.667 lb	26.674 lb	26.648 lb	-26.660 lb

ANALYTICAL SOLUTION:

Using Donnell Approximations.

$$P_{cr} = 4EI / R^3 = 26.667 \text{ lb/in}$$

Figure B5-1



Problem Sketch   Finite Element Model

B6: Buckling Analysis of a Small Frame

TYPE:

Buckling analysis, truss (TRUSS2D) and beam (BEAM3D) elements.

REFERENCE:

Timoshenko, S. P., and Gere, J. M., “Theory of Elastic Stability,” 2nd ed., McGraw-Hill Book Co., New York, 1961, p. 45.

GIVEN:

$L = 20$ in

$A_B = 4$ in^2

$A_T = 0.1$ in^2

$E = E_B = E_T = 30E6$ psi

$I_B = 2$ in^4

COMPARISON OF RESULTS:

	Theory	COSMOS/M
P_{cr1}	1051.392 lb	1051.367 lb
P_{cr2}	1480.44 lb	1481.20 lb

ANALYTICAL SOLUTION:

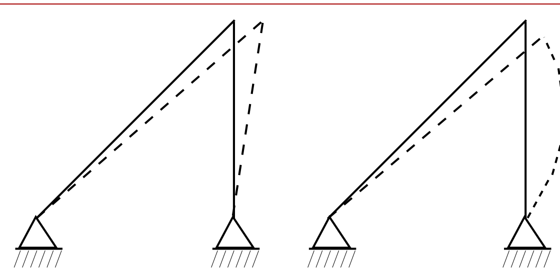
The classical results are obtained from:

$$P_{1cr} = A_T E \operatorname{Sin} \alpha \operatorname{Cos}^2\alpha \,/\, (1 + (A_T/A_B) \operatorname{Sin}^3\alpha)$$

$$P_{2cr} = \pi^2 E I_B \,/\, L^2$$

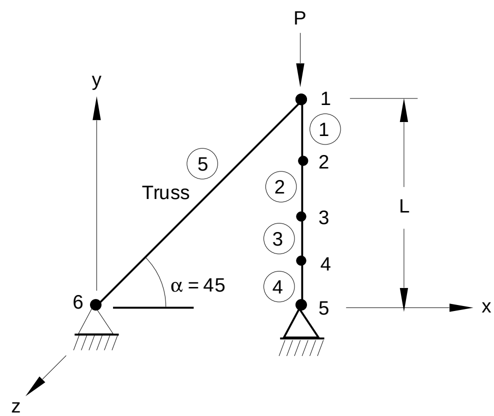
MODE SHAPES:

Figure B6-1



Mode Shape 1 Mode Shape 2

Figure B6-2



Problem Sketch and Finite Element Model

B7A, B7B: Instability of Frames

TYPE:

Buckling analysis, shell element (SHELL4 and SHELL6).

REFERENCE:

Brush, D. O., and Almroth, B. O., “Buckling of Bars, Plates, and Shells,” McGraw-Hill, Inc., New York, 1975, p.29.

PROBLEM:

Find the buckling load and deflection mode for the frame shown below.

GIVEN:

$E = 30 \times 10^6$ psi
$h = 1$ in
$L = 25$ in
$I = 1/12 \text{ in}^4$

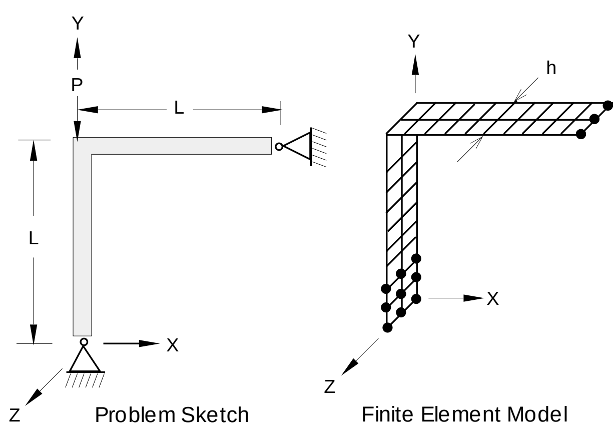
COMPARISON OF RESULTS:

	Theory	COSMOS/M		
		SHELL4	SHELL6 (Curved)	SHELL6 (Assembled)
P_{cr}	55506.6 lb	56280.8 lb	55364.9 lb	55732.1lb

ANALYTICAL SOLUTION:

$$P_{cr} = 1.406\pi^2 EI / L^2 = 55506.6 \text{ lb}$$

Figure B7-1



B8: Instability of a Cylinder

TYPE:

Buckling analysis, axisymmetric shell element (SHELLAX).

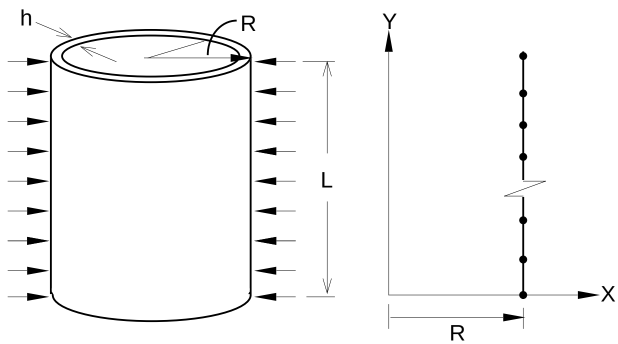
REFERENCE:

Brush, D. O., and Almroth, B. O., “Buckling of Bars, Plates, and Shells,” McGraw-Hill, Inc., New York, 1975, p. 164.

PROBLEM:

Find the buckling load and deflection mode for a cylindrical shell that is simply supported at its ends and subjected to uniform lateral pressure.

Figure B8-1



Problem Sketch

Finite Element Model

GIVEN:

E = 10×10^6 psi

h = 0.2 in

R = 20 in

L = 20 in

ν = 0.3

COMPARISON OF RESULTS:

	Theory	COSMOS/M
P_{cr}	106 psi	113.97 psi

ANALYTICAL SOLUTION:

$$P_{cr} = \frac{Eh}{R}\left\{\left[\frac{(\pi R/L)^2 + n^2}{n}\right]^2 \times \frac{(h/R)^2}{12(1-\nu^2)} + \frac{(R/L)^4}{n^2\left[\left(\frac{\pi R}{L}\right)^2 + n^2\right]}\right\}$$

B9: Simply Supported Stiffened Plate

TYPE:

Buckling analysis, shell (SHELL4) and beam (BEAM3D) elements.

REFERENCE:

Timoshenko, S. P., and Gere, J. M., “Theory of Elastic Stability,” 2nd edition, McGraw-Hill Book Co., Inc., New York, p. 394, Table 9-16.

PROBLEM:

A simply supported rectangular plate is stiffened by a beam of rectangular cross-section as shown in the figure. The stiffened plate is subjected to inplane pressure at edges x = 0 and x = a. Determine the buckling pressure load.

GIVEN:

$E = 30{,}000\ \text{kip/in}^2$

$\nu = 0$

$h_p = 1$ in

$a = 45.5$ in

$b = 42$ in

$b_b = 0.42$ in

$h_b = 10$ in

ANALYTICAL SOLUTION:

$$\sigma_{cr} = \frac{\pi^2 D}{b^2 h} \times \frac{\left(1+\beta^2\right)^2 + 2\gamma}{\beta^2\left(1+2\delta\right)}$$

Where:

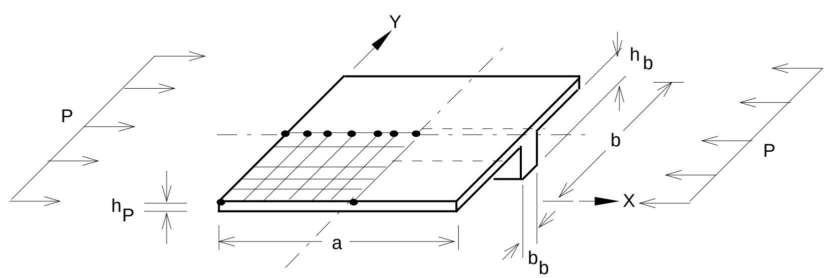
$\beta = a/b \quad \gamma = EI_b / bD \quad D = E(h_p)^3 / 12(1-\nu^2)$

$A_b = b_b \times h_b \quad \delta = A_b / bh_p$

COMPARISON OF RESULTS:

	Theory	COSMOS/M	Difference
P_{cr}	223.80 kip/in	232.53 kip/in	3.9%

Figure B9-1



Problem Sketch and Finite Element Model

B10: Stability of a Rectangular Frame

TYPE:

Buckling analysis, beam elements (BEAM2D).

REFERENCE:

Timoshenko, S. P. and Gere J. M., "Theory of Elastic Stability," McGraw-Hill Book Co., New York, 1961.

GIVEN:

L = b = l00 in

A = 1 in^2

h = 1 in (beam cross section height)

I = 0.0833 in^4

E = 1 x 10^7 psi

P = 100 lb

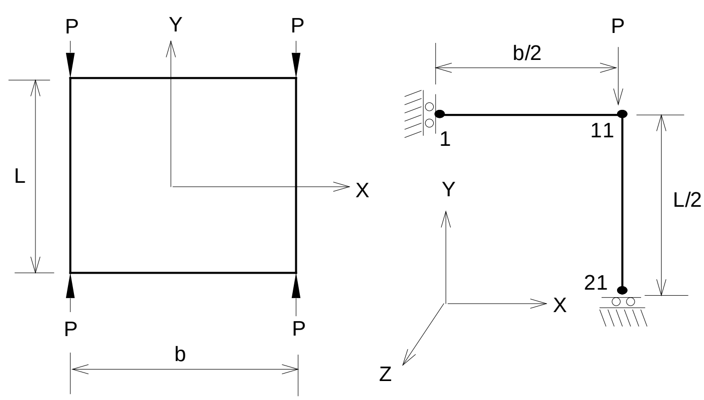
COMPARISON OF RESULTS:

	Theory	COSMOS/M
P_{cr}	1372.45 lb	1371.95 lb

ANALYTICAL SOLUTION:

$$P_{cr} = 16.47EI/L^2 = 1372.4451 \text{ lb}$$

Figure B10-1



Problem Sketch   Finite Element Model

B11: Buckling of a Stepped Column

TYPE:

Buckling analysis, beam element (BEAM2D).

RFERENCE:

Roark, R. J. and Young, Y. C., “Formulas for Stress and Strain,” McGraw-Hill, New York, 1975, pp. 534.

PROBLEM:

Find the critical load and mode shape for the stepped column shown below.

GIVEN:

$L = 1000$ mm
$A_1 = 10{,}954$ mm^2
$A_2 = 15{,}492$ mm^2
$E_1 = E_2 = 68{,}950$ MPa
$\nu_1 = \nu_2 = 0.3$
$I_1 = 1 \times 10^7$ mm^4
$I_2 = 2 \times 10^7$ mm^4
$P_1/P_2 = 0.5$

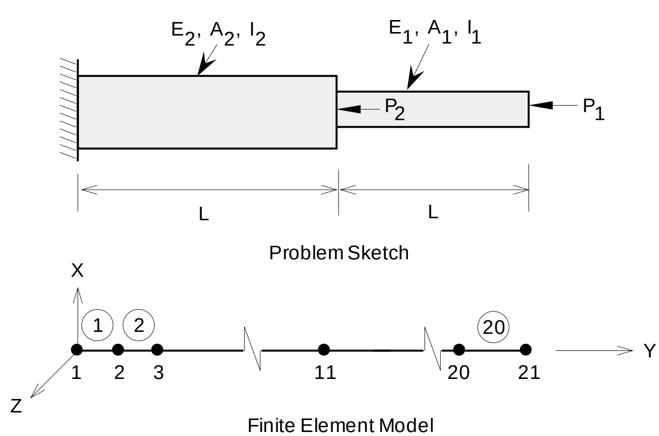
COMPARISON OF RESULTS:

	Theory	COSMOS/M
P_{cr}	554.6 KN	554.51 KN

ANALYTICAL SOLUTION:

$$P_{cr} = 0.326\pi^2 E_1 I_1 / (2L)^2 = 554{,}600 \text{ N}$$
$$= 554.6 \text{ KN}$$

Figure B11-1



B12: Buckling Analysis of a Simply Supported Composite Plate

TYPE:

Buckling analysis, composite shell element (SHELL4L).

REFERENCE:

Jones, "Mechanics of Composite Material," McGraw-Hill Book Co., New York, p. 269.

PROBLEM:

Find the buckling load for [45,-45,45,-45] antisymmetric angle-ply laminated plate under uniform axial compression p.

GIVEN:

a = b = 40 in

h = Σh_i = 1 in

E_x = 400,000 psi

E_y = 10,000 psi

ν_{xy} = 0.25

$G_{xy} = G_{yz} = G_{xz}$ = 5,000 psi

p = 1 lb/in^2

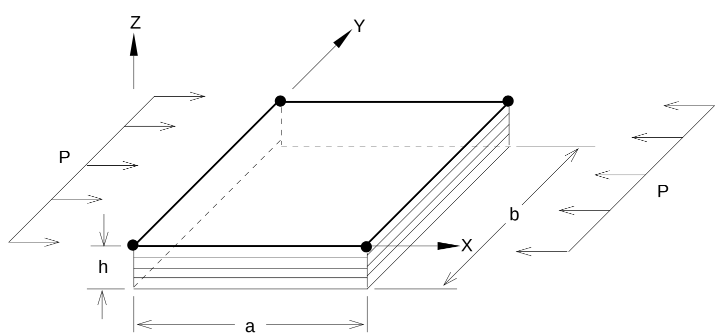
COMPARISON OF RESULTS:

	Theory	COSMOS/M
P_{cr}	334.0 lb/in	345.36 lb/in

ANALYTICAL SOLUTION:

Approximate solution is given by graph 5-16 in the reference.

Figure B12-1



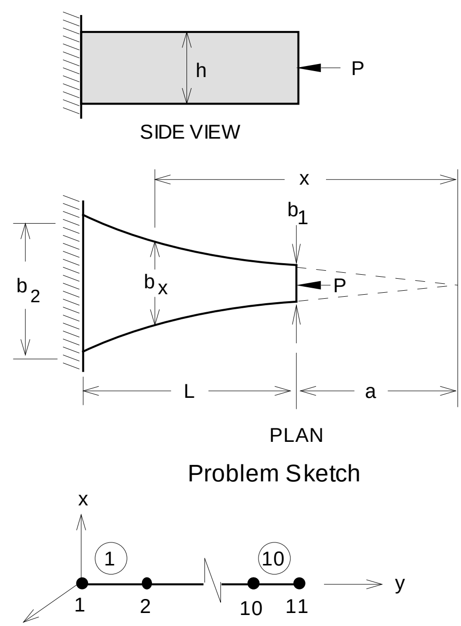
Problem Sketch and Finite Element Model

B13: Buckling of a Tapered Column

TYPE:

Buckling analysis, beam element (BE

Figure B13-1



Problem Sketch

Finite Element Model

REFERENCE:

Timoshenko, S. P., and Gere, J. M., "
Book Co., New York, 1961, pp. 125-1

GIVEN:

b_1 = 1 in
b_2 = 4 in
b/b_1 = $(x/a)^2$
I_1 = 1 in^4
I_2 = 4 in^4
I_1/I_2 = 0.25
L = 100 in
a = 100 in
E = 1×10^7 psi
h = 1 in

ANALYTICAL SOLUTION:

$$P_{cr} = 1.678EI_2 / L^2 = 6712 \text{ lb}$$

COMPARISON OF RESULTS:

The Critical Load:

	Theory	COSMOS/M
P_{cr}	6712 lb	6718 lb

B14: Buckling of Clamped Cylindrical Shell Under External Pressure Using the Nonaxisymmetric Buckling Mode Option

TYPE:

Linear buckling analysis using the nonaxisymmetric buckling mode option (SHELLAX).

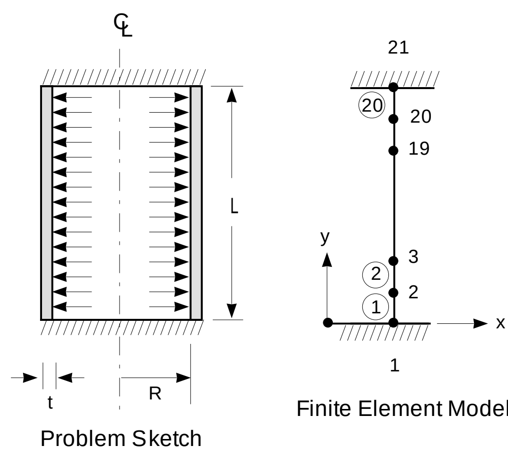
REFERENCE:

Sobel, L. H., “Effect of Boundary Conditions on the Stability of Cylinders Subject to Lateral and Axial Pressures,” AIAA Journal, Vol. 2, No. 8, August, 1964, pp 1437-1440.

PROBLEM:

Find the buckling pressure for the shown axisymmetric clamped-clamped shell.

Figure B14-1



GIVEN:

R = 1 in

ν = 0.3

L = 4 in

E = 10^7 psi

t = 0.01 in

MODELING HINTS:

The cylindrical shell is modeled with 20 uniform elements. The starting harmonic number for which the buckling load is calculated is set to 2. The minimum buckling load occurs at harmonic number 5 which corresponds to mode shape 4 since the program started from harmonic 2.

COMPARISON OF RESULTS:

	Theory	COSMOS/M
Harmonic Number	5	5
Critical Load	33.5 psi	35.0 psi

B15A, B15B: Buckling of Simply-Supported Cylindrical Shell Under Axial Load

TYPE:

Linear buckling analysis using the nonaxisymmetric buckling mode option (SHELLAX).

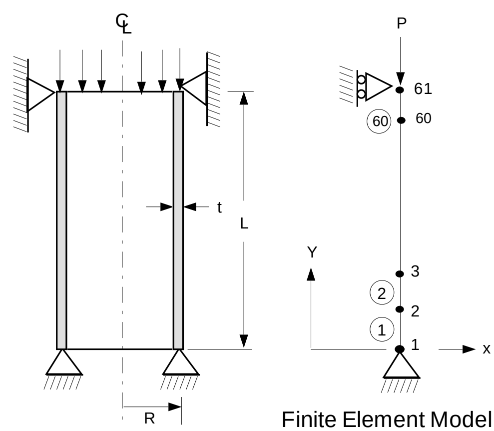
REFERENCE:

Timoshenko, S. P., and Gere, J. M., “Theory of Elastic Stability,” McGraw-Hill Book Co., 1961.

PROBLEM:

Find the buckling load for the simply-supported cylindrical shell shown in the figure below.

Figure B15A and B15B



Problem Sketch

Finite Element Model

GIVEN:

R	= 10 in
L	= 16 in
ν (NUXY)	= 0.3
E	= 10^7 psi
t	= 0.1 in

MODELING HINTS:

The cylindrical shell was modeled with 60 uniform elements. The starting harmonic number for which the buckling load is calculated was set to 1. The solution stopped at harmonic number 2 at which the minimum buckling load occurs. The number of maximum iterations for the eigenvalue calculations was set to 100.

COMPARISON OF RESULTS:

	Harmonic No.	Critical Load	
		B15A (SHELLAX)	B15B (PLANE2D)
Theory	2	6.05×10^4 lb/rad	6.05×10^4 lb/rad
COSMOS/M	2	6.07×10^4 lb/rad	6.02×10^4 lb/rad

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